Students' Algebraic Thinking in Using the Geogebra Application

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Abstract. Algebra is knowledge in education. The ability to think should be known so that it can be maximized in the learning process, one of which is the ability to think in algebra. In the process of working on various algebra problems, students carry out generational activities, transformation activities, and global meta-level activities. Learning algebra will be easy if combined with technology. One of the technologies that can support algebra learning is Geogebra. The research aims to determine how students' algebraic thinking abilities use the Geogebra application. The method in this research is qualitative, with a case study type. The research subjects were students taking calculus courses in the Informatics Engineering study program at a private campus in Jakarta. The research results show that the algebraic thinking ability levels consist of high, medium, and low. High algebraic thinking abilities have high generational, transformational, and global meta-level abilities, while moderate algebraic abilities have moderate generational abilities; transformational abilities tend to be medium to high, and global meta-level abilities tend to be medium to low. Low levels of algebraic thinking ability tend to have moderate to low generational abilities, low transformational abilities, and low global meta-level abilities.

Keywords: Algebraic thinking, students, GeoGebra, generational, transformational, global meta-level

INTRODUCTION

Algebra is knowledge in education. According to Hodgen et al. (2018), algebra is a cultural artifact - a body of knowledge embedded in educational systems worldwide. According to Kaput (2017), algebra is a cultural product, a body of knowledge embedded in the world's education system.

The thinking process is the activity of receiving, processing, understanding, and identifying information, as well as remembering and combining knowledge to achieve a specific understanding. Algebraic thinking uses symbols and mathematical tools to represent information mathematically in words, diagrams, tables, graphs, and equations and to analyze various situations, such as testing evidence, finding unknown values, and searching for evidence. (Ntsohi, 2013).
Algebraic thinking is also a human activity from which algebra emerged (Kaput, 2017).

The ability to think should be known so that it can be maximized in the learning process, encouraging algebra abilities to the maximum that students can do with their experience and abilities (Masnia et al., 2022). One is the ability to think algebraically (Kieran, 2004). In the process of working on algebra problems, students carry out generational activities, transformational activities, and global meta-level activities. In line with Wilkie (2016), algebraic thinking is an ability that consists of three primary skills, namely the ability to use algebraic symbols and relationships, use multiple representations (such as symbolic, graphical, and tabular), and formulate generalizations. As shown by research conducted by Jupri (2015) and Şengül & Erdoğan (2014), students do not understand basic algebra concepts such as variables and equations. Likewise, research by Farida & Hakim (2021) and Töman & Gökburun (2022) showed that students' algebraic thinking abilities are still low.

Learning algebra will be easy if combined with technology. Applications are more effective in learning mathematics (Masnia et al., 2020). Education and new technologies can make mathematics education easier (Weinhandl, 2020). Applications are a technology that makes it easier for students to learn. One application that is often used for algebra is geometry. "Geogebra" mathematics software combines interactive geometry, algebra, statistics, and calculus to create the most comprehensive application for generating and explaining mathematical ideas for elementary and university-level students (Dahal et al., 2019).

When technology is integrated into education, the focus should be on the learning process and students rather than technology. Research shows a difference in average learning outcomes between classes that use the Geogebra application and those that do not (Bedada, 2022). The results of learning using the Geogebra application have a good effect (Apriani & Hayati, 2022).

Based on background, this research focuses on how generational, transformational, and global meta-level abilities are based on medium and low-level algebraic thinking abilities.
RESEARCH METHODS

This research method uses a qualitative approach with a case study type. Research subjects were selected based on purposive sampling. Sukestiyarno (2021) Purposive sampling is a non-probability sampling method where researchers use their judgment to select cases or events that can provide the best information to achieve research objectives. The population of this study was 300 students divided into seven classes. With purposive sampling, researchers used one class with 28 students from the Informatics Engineering study program at one of Jakarta's private campuses. The instrument used in this research tested algebraic thinking ability through descriptions and in-depth interviews. The results of algebraic thinking ability were divided into three categories: high, medium, and low. From each category, two students were selected. In data triangulation, thinking ability the author also conducted in-depth interviews for research based on the thinking ability test category. The results of the description test were analyzed based on the following indicators: (1) generational activities, (2) transformational activities, and (3) global meta-level activities.

RESULTS AND DISCUSSION

algebraic thinking ability, moderate algebraic thinking ability, and low algebraic thinking ability. The following is the Algebra ability data that the author obtained in Table 1.

<table>
<thead>
<tr>
<th>Algebraic Thinking Ability</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tall</td>
<td>7</td>
</tr>
<tr>
<td>Currently</td>
<td>12</td>
</tr>
<tr>
<td>Low</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 1. Algebraic Thinking Ability Groups

Based on the data in Table 1, data was then triangulated based on categories of algebraic thinking ability, and two students were selected from each category.

A. High Algebra Ability

1) Subject AT student
Based on Figure 1, subject AT answered correctly, but performing the function derivative more than once should have been done only once. The AT subject is included in the category of high algebraic thinking ability. In generational activities, AT subjects understand the emerging generalizations, understand the variables in the problem, and present the problem by making connections between variables.

AT subjects can determine equivalent algebraic forms in transformational activities, carry out algebraic operations, and solve equations. AT subjects are included in high transformational activities. AT subjects' Global meta-level activities can apply algebra in analyzing changes and relationships and predicting problems in mathematics; AT subjects are included in high global meta-levels.

The data triangulation process for AT subjects was carried out through interviews as follows:

Q: Are the questions given understandable?
AT: Understandable
Q: What information did you get from the questions given?
AT: about increasing derivatives and maximum points, apart from that about factoring and derivatives
Q: Explain the meaning of the variables in the problem!
AT: t is the period of virus growth
Q: How do you understand the relationship between the variables in the problem?
AT: by studying the function given in the question
Q: Please explain the meaning of your answer.
AT: Generally, a function uses the symbol f(x) so we can change variables and understand the contents of the problem.
Q: Explain the algebraic form that is equivalent to the problem given!
AT: the same as the first problem about viruses f(t) = c + t + t^n
Q: How do you perform algebraic operations on the problems given?
AT : uses the formula that has been taught
Q : How did you get that answer? Explain the steps you used to solve this equation.
AT : first, use a derivative formula, then factor it to find specific points, then draw a graph and determine the graph results (positive and negative)
Q : What changes have occurred in the questions?
AT : changes that occur when the problem is revealed can be solved
Q : How did you solve the problem?
AT : using the formulas given by the lecturer and looking at the internet as a reference for answers

Based on the results of interviews, AT subjects understand the information in the questions given, the variables and their relationships with other variables, and the meaning of the answers given. AT subjects also understand algebraic forms, operate them, and can solve the equations given in the problem. Apart from being able to solve the problem as a whole, the AT subject can also understand the changes that occur in the problem.

The instruments given to AT subjects are in the form of interviews or assessments of students’ understanding of algebraic concepts. Subjects called AT demonstrated high-level algebraic thinking abilities based on interview responses. AT shows the meaning of the problem given, the definition of variables, the relationship between variables, and the algebraic form of the problem equation. Apart from that, AT can operate and solve algebraic equations, understand changes that occur in the problem, and explain the steps taken to solve the problem. The interview results also showed that AT used formulas taught by teachers and internet references to solve problems.

The interview reflects AT's skills in understanding and applying algebraic concepts and the ability to solve problems using algebraic methods. His responses demonstrated a strong understanding of algebraic thinking and problem-solving skills.

2) GDS Student Subject

Based on Figure 2, GDS subjects answered the questions in order and correctly. The GDS subject is included in the category of high algebraic thinking ability. GDS subjects can engage in generational activities, namely understanding generalizations that arise, understanding variables, and
presenting problems by connecting variables. DGS subjects are included in the high generational category.

![Figure 2. GDS Subject Answers](image)

Rational genes' algebraic ability indicators are transformational. In transformational activities, GDS subjects can determine equivalent algebraic forms, carry out algebraic operations, and solve them in an equation. GDS subjects are considered highly transformational.

The next activity is a global meta-level activity. In this activity, GDS subjects can apply algebra, analyze, connect, and predict problems in mathematics. GDS subjects are included in the high global meta-level

The data triangulation process on GDS subjects was carried out through interviews as follows:

\[ Q : Are \ the \ questions \ given \ understandable? \]
\[ GDS : Yes, \ partly \]
\[ Q : What \ information \ did \ you \ get \ from \ the \ questions \ given? \]
\[ GDS : The \ information \ I \ got \ was \ from \ this \ question \ using \ a \ derivative \ formula \]
\[ Q : Explain \ the \ meaning \ of \ the \ variables \ in \ the \ problem? \]
\[ GDS : In \ question \ number \ 1, \ the \ variable \ (t) \ is \ the \ period \ of \ virus \ growth, \ and \ in \ question \ number \ 2, \ the \ variable \ (x) \ is \ the \ unknown \ height \ of \ the \ square \]
\[ Q : How \ do \ you \ understand \ the \ relationship \ between \ the \ variables \ in \ the \ problem? \]
\[ GDS : The \ relationship \ between \ these \ variables \ is \ that \ they \ both \ look \ for \ the \ maximum \ value. \]
\[ Q : Please \ explain \ the \ meaning \ of \ your \ answer. \]
\[ GDS : The \ meaning \ of \ the \ answer \ to \ question \ number \ 1 \ is \ about \ the \ range \ of \ decreasing, \ increasing, \ and \ maximum \ levels \ of \ virus \ growth. \]
\[ P : Explain \ the \ algebraic \ form \ that \ is \ equivalent \ to \ the \ problem \ given! \]
\[ GDS : available \ at \ 60t - 3t^2 \]
\[ Q : How \ do \ you \ perform \ algebraic \ operations \ on \ the \ problems \ given? \]
GDS: The method is to operate numbers with the same variables by adding, subtracting, multiplying, and dividing

Q: How did you get that answer? Explain the steps you used to solve this equation.

GDS: You have to look for its function first and then look for the period for the development of the virus

Q: What changes have occurred in the questions?

GDS: The changes that occur in the problem are after we look for unknown variables

Q: How did you solve the problem?

GDS: The method is to use the derivative of each function in the problem

Based on the results of interviews, the GDS subject understands the questions, the variables and their relationships with other variables, and the meaning of the answers. GDS subjects also understand algebra, its operations, and solving equations in problems. Apart from that, GDS subjects can also solve the questions as a whole and understand the changes in the questions.

Based on interview data and tests of algebraic thinking abilities, it appears that GDS subjects demonstrate high-level algebraic thinking abilities. The interview responses showed that GDS understood the questions, the meaning of variables, the relationships between variables, and the algebraic form of the question equations. GDS can also operate and solve algebraic equations, understand changes in the problem, and explain the steps taken to solve the problem. The results of this search further support the GDS assessment of high-level algebraic thinking abilities because they discuss indicators of generational, transformational, and global algebraic thinking abilities in algebraic thinking.

Interviews and algebraic thinking ability test results show GDS's skills in understanding and applying algebraic concepts and the ability to solve problems using algebraic methods. Results demonstrate a strong understanding of algebraic thinking and algebraic problem-solving skills.

B. Moderate algebraic ability

1) AF Student Subject
Based on Figure 3, subject AF answered the question with the first step of deriving the function, determining the value of $t$. However, in answering the question it was not answered in detail; subject AF only answered the final result. The author considers that subject AF's answer is included in the moderate algebraic thinking ability category. AF subjects can engage in generational activities, namely understanding generalizations that arise, understanding variables, and presenting problems by making connections between variables, but not until they are finished with the fact that the question is not solved correctly. AF subjects fall into the medium generational category.

The next indicator of algebraic ability is transformational. Transformational activities of AF subjects can determine equivalent algebraic forms, perform algebraic operations, and solve equations; AF subjects are considered highly transformational. Global meta-level activity AF subjects can apply algebra. Less able to analyze, connect, and predict problems in mathematics, AF subjects are included in the medium global meta-level.

The data triangulation process on AF subjects was carried out through interviews as follows:

$Q$ : Are the questions given understandable?
$AF$ : Yes

$Q$ : What information did you get from the questions given?
$AF$ : A function

$Q$ : Explain the meaning of the variables in the problem!
$AF$ : variable to determine time

$Q$ : How do you understand the relationship between the variables in the problem?
$AF$ : $d$ I understand the problem

$Q$ : Please explain the meaning of your answer.
$AF$ : determine the function first; determine the $f, f'$
Based on the interview results, AF subjects understood the questions given, the variables, and their relationships with other variables but did not understand the meaning of the answers. AF subjects also understand algebraic forms and their operations but are less able to solve equations in problems. Apart from that, AF subjects were also less able to solve the questions as a whole and less able to understand the changes in the questions.

Based on interview data and the given algebraic thinking ability test results, subject AF shows a moderate algebraic thinking ability. The interview responses show that AF understands the problems, the meaning of variables, and the relationships between variables. However, AF's response lacked detail and did not fully answer the question. AF can determine the equivalent algebraic form of the problem, operate on the algebraic equation, and solve it. However, AF's ability to analyze, relate, and predict problems using algebraic methods is limited. The results of this search further support AF's assessment of moderate-level algebraic thinking ability because it discusses generational, transformational, and global algebraic thinking ability indicators.

The results of interviews and algebraic thinking ability tests show AF's proficiency in understanding and applying algebraic concepts and the ability to solve problems using algebraic methods. However, AF's ability to analyze, relate, and predict problems using algebraic methods is limited.
2) RMA Student Subject

![Figure 4. RMA Subject Answers](image)

Based on Figure 4, the RMA subject answered the question with the first step of deriving the function and determining the t value, but the steps in answering the question were not detailed, and only the final answer was written. The author believes that the RMA subject's answers fall into moderate algebraic thinking ability. RMA subjects can engage in generational activities, namely understanding emerging generalizations, understanding variables, and presenting problems, but are less able to make connections between variables. RMA subjects are in the moderate generalization category.

The RMA subject's transformational activities can determine equivalent algebraic forms and perform algebraic operations but cannot solve equations. The RMA subject is categorized as medium transformational. The global meta-level activity of RMA subjects is lacking in applying algebra and is less able to analyze, connect, and predict problems in mathematics. AF subjects are included in the low global meta-level.

The process of data triangulation on RMA subjects was carried out through interviews as follows:

The following is an excerpt from the RMA Subject's interview

\[ Q \] : Are the questions given understandable?
\[ RMA \] : Understandable

\[ Q \] : What information did you get from the questions given?
\[ RMA \] : How to calculate using the f(x) function
Q: Explain the meaning of the variables in the problem!
RMA: Looks for conditions where the value increases and decreases, as well as when the value reaches a maximum point.
Q: How do you understand the relationship between the variables in the problem?
RMA: If you already know the value
Q: Please explain the meaning of your answer.
RMA: I solved the problem by using the derivative function to get the maximum and minimum values.

P: Explain the algebraic form that is equivalent to the problem given!
RMA: \( F(t) = 1000 + 30t^2 - t^3 \)
Q: How do you perform algebraic operations on the problems given?
RMA: By derivative method
Q: How did you get that answer? Explain the steps you used to solve this equation.
RMA: By calculating using the function derivative method
Q: What changes have occurred in the questions?
RMA: The changes can be completed with simple results
Q: How did you solve the problem?
RMA: By using function derivatives

Based on the interview results, the RMA subjects showed a reasonably good understanding of the questions given, the variables, and the relationships between variables. RMA subjects can also explain the meaning of their answers and understand algebraic forms and their operations. However, RMA subjects are less able to solve equations in detail and understand the changes in the problem. It shows that the algebraic thinking abilities of RMA subjects tend to be in the medium category, especially in generational and transformational activities. However, RMA subjects still need to improve their abilities in global meta-level activities, especially in applying algebra and analyzing and predicting problems in mathematics.

RMA subjects apply algebraic thinking skills in solving problems, with the first step being deriving the function and determining the t value. RMA subjects understand the meaning of the variables in the problem, namely looking for conditions where the value increases and decreases and when the value reaches a maximum point. RMA subjects can also determine an algebraic form equivalent to the problem given and perform algebraic operations by deriving functions.
However, RMA subjects were less able to solve the equation as a whole and understand the changes in the problem.

C. Low Algebra Ability

1) EDA student subject

![Image: EDA Subject Answers]

**Figure 5. EDA Subject Answers**

Based on Figure 5, the EDA subject answered the question with the first step of deriving the function, not determining the t value, and answering only one question; the second question was only answered modestly, and the third question was not. The author believes that the EDA subject's answers fall into the low algebraic thinking ability category. EDA subjects can engage in generational activities, namely understanding generalizations emerging and understanding variables, but are less able to present problems and make connections between variables. EDA subjects are in the moderate generalization category.

EDA subjects' transformational activities can determine equivalent algebraic forms; carrying out algebraic operations is not yet capable of solving equations; EDA subjects are low transformational. Global meta-level activity EDA subjects are not yet able to apply algebra and analyze, connect, and predict problems in mathematics. EDA subjects are included in the low global meta-level.

The data triangulation process on the EDA subject was carried out through interviews as follows:

Following are EDA Subject interview excerpts

**Q**: Are the questions given understandable?
**EDA**: Somewhat less understanding

**Q**: What information did you get from the questions given?
**EDA**: How to calculate the maximum and minimum measurements using the f(x) function

**Q**: Explain the meaning of the variables in the problem!
**EDA**: Several things must be looked for first using other variables
Q : How do you understand the relationship between the variables in the problem?
EDA : Using algebraic concepts
Q : Please explain the meaning of your answer.
EDA : Because it makes it easier to solve variable problems
P : Explain the algebraic form that is equivalent to the problem given!
EDA : There are variables x and y as well as constants
Q : How do you perform algebraic operations on the problems given?
EDA : By grouping
Q : How did you get that answer? Explain the steps you used to solve this equation.
EDA : First of all, let us group what we have obtained. Then, look for the variable, reduce it, and look for the maximum or minimum point.
Q : What changes have occurred in the questions?
EDA : The change after lowering the variable value changes
Q : How did you solve the problem?
EDA : Looking for formulas from the internet

Based on the interview results, the EDA subject showed a poor understanding of the questions given, the variables, and the relationship between variables. EDA subjects are also less able to explain the meaning of the variables contained in the problem and less able to present the problem and make connections between variables. In addition, EDA subjects are less capable of transformational activities such as solving equations and less capable of global meta-level activities such as applying algebra, analyzing, and predicting problems in mathematics. It shows that the algebraic thinking abilities of EDA subjects tend to be in the low category.

Low algebraic thinking ability in the EDA subject refers to a lack of understanding and application of algebraic concepts in solving mathematical problems. EDA subjects cannot yet understand the problem as a whole, including variables and relationships between variables. In addition, EDA subjects are less capable of transformational activities, such as solving equations, and global meta-level activities, such as applying algebra, analyzing, and predicting problems in mathematics. It shows that EDA subjects do not have adequate abilities for applying algebraic concepts in the context of mathematical problems.
Based on Figure 6, subject MYH answered the question by performing the first step of deriving the function. According to the author's perspective, subject MYH lacked an understanding of the function's origin. Concerning the t value, subject MYH merely transcribed without grasping the content. When responding to the question, subject MYH provided a single answer devoid of elaboration; furthermore, they neglected to address the subsequent question. The author concludes that the response from subject MYH indicates a deficiency in algebraic thinking ability, categorizing it as low.

MYH subjects can engage in generational activities, namely understanding generalizations that arise, being unable to understand variables, presenting problems, and not being able to make connections between variables. MYH subjects are in the low generalization category.

MYH subject's transformational activity cannot yet determine the equivalent algebraic form; it cannot carry out algebraic operations in solving equations. MYH subject is categorized as low transformational. Global meta-level activity MYH subjects cannot apply algebra, analyze, connect, and predict problems in mathematics. MYH subjects are included in the low global meta-level.

The data triangulation process on the MYH subject was carried out through interviews as follows:

The following is an excerpt from MYH Subject's interview

Q : Are the questions given understandable?
MYH : Little Understood

Q : What information did you get from the questions given?
MYH : Logical thinking from algebra
Based on the interview results, MYH subjects did not understand the questions given, the variables and their relationships with other variables, or the meaning of the answers. MYH subjects also do not understand algebraic forms and their operations and cannot solve equations in problems. MYH subjects could also not solve the questions entirely and could not understand the changes in the questions.

Based on the analysis, the author concluded that the MYH subject's answers were included in the low algebraic thinking ability category. Subjects have low abilities in generational, transformational, and global meta-level activities. Subjects have difficulty understanding problems, variables, and relationships between variables. Subjects also have difficulty understanding algebraic forms operations, and solving equations. Apart from that, the subject cannot solve the problem as a whole and does not understand the changes in the problem. The interview results showed that the subject had difficulty understanding problems, variables, and relationships between variables. The subject also had difficulty understanding the meaning of the answer and its algebraic form. The subject cannot solve the problem and does not understand the changes in the problem.
From research subjects, the author can describe algebraic thinking abilities in Table 2.

<table>
<thead>
<tr>
<th>Subject</th>
<th>Generational</th>
<th>Transformational</th>
<th>Global meta-level</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT</td>
<td>Tall</td>
<td>Ring high</td>
<td>Tall</td>
</tr>
<tr>
<td>GDS</td>
<td>Tall</td>
<td>Tall</td>
<td>Tall</td>
</tr>
<tr>
<td>AF</td>
<td>Currently</td>
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<td>Currently</td>
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<td>Currently</td>
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<tr>
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<td>Render</td>
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<tr>
<td>MY</td>
<td>Low</td>
<td>Low</td>
<td>Low</td>
</tr>
</tbody>
</table>

Table 2. Algebraic Thinking Ability

Research findings show that cognitive abilities in algebra vary. For example, AT has high levels of generative and meta-global capabilities but low levels of transformational capabilities. On the other hand, GDS has high capabilities in these three categories. AF has moderate generative and meta-global capabilities but high transformational capabilities. RMA has moderate ability in generative and transformational activities but low meta-global ability. EDA has moderate generative capabilities but low transformational and meta-global capabilities. MYH has low capabilities in these three categories.

Based on research, cognitive abilities in algebra vary among students. It is in line with the research results of Badawi et al. (2016), showing that students in the high-level group generally have high generative, transformational, and meta-global abilities, while students in the low-level group have lower abilities in these areas. The second study focused on eighth-grade students’ generative, transformational, and meta-global abilities. The findings show that students who are in the high-level category tend to have higher abilities in these areas. In comparison, students who are in the low-level category show lower abilities (Misbahuddin et al., 2019). These relevant research results provide empirical evidence of the wide variety of cognitive abilities in algebra among students, supporting the idea that different students have different levels of generative, transformational, and meta-global abilities in the context of algebraic thinking.
These findings can be helpful for educators and researchers to develop strategies to improve algebraic cognitive abilities. For example, for low transformational abilities, such as AT, educators can focus on developing activities that increase transformational abilities. Educators can focus on developing activities that increase meta-global abilities for low meta-global abilities, such as EDA. By understanding cognitive abilities in algebra, educators can adapt their teaching strategies to support student learning better.

CONCLUSION

Algebraic thinking in using the Geogebra application with subjects with high algebraic thinking abilities tend to have high generational, transformational, and global meta-level abilities. Subjects with moderate algebraic thinking abilities have moderate generational abilities, moderate to high transformational abilities, and moderate to low global meta-level abilities. Subjects with low abilities tend to have low to moderate generational abilities, low transformational abilities, and relatively low global meta-level abilities.

REFERENCES


