

Students' Analytical Thinking in Solving Problems of Polygon Areas

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Abstract. Elementary students' understanding of polygon areas concept is important in succeeding their academic and daily life because the concept is broadly applied at schools and homes. Hence, comprehensive understanding of the concept is required. The understanding can be seen from students' analytical thinking in facing a complicated problem. If a student does analytical thinking, the student can create a link between the concepts and predict what will happen. In fact, students frequently use the procedural thinking to solve almost any type of problems, including non-routine problems. A study to reveal this phenomenon is thus important to conduct. This study aimed to describe students' analytical thinking in solving the polygon areas problems. To know the students' analytical thinking, the researcher gave problems to a team of mathematics Olympiad of elementary school students. Based on students' analytical answers, the researcher found (1) analytical and (2) semi-analytical thinking. Analytical thinking was characterized by algorithm clarity, chronological reasoning, valid argumentation, and effective steps. Semi-analytical thinking was characterized by a presence of "disturbing elements" which broke the chain of implications. The result of the study can be teachers' consideration in selecting teaching methods tailored to the students' thinking possibilities so that knowledge and learning experiences are well internalized.

Keywords: Analytical thinking, problem-solving, polygon areas

INTRODUCTION

Mathematics is formed empirically through human experiences. Those experiences are processed rationally and analytically with reasoning in cognitive structure, so that mathematical concepts can be formed. (Windsor, 2008) states that studying mathematics is like thinking about the patterns, communicating the patterns, or learning the patterns. Studying mathematics means learning about something abstract and containing insight encoded to symbols and figures. Hence, studying mathematics requires students to have thinking skills and reasoning.

Analytical thinking is one of thinking models that needs to be developed in studying mathematics because the objects of mathematics are something abstract (Parta, 2016). It is important not only to teach students about facts but also to teach them to think analytically, creatively, practically, and wisely. Some researches show that analytical thinking is correlated positively with students' academic achievements (Dunn et al., 2010; Greene, Miller, Crowson, Duke, &

Akey, 2004; Kuhn & Holling, 2009; Lopez & Tancinco, 2016; Parta, 2016; Sudiby, Jatmiko, & Widodo, 2016; Taleb, 2016; Thaneerananon, Wannapong, & Nokkaew, 2016; Zhang, 2005). Based on those experts, it can be concluded that analytical thinking should be developed from the students' perspectives.

Analytical thinking includes the abilities of differentiating and categorizing elements from events or things with the purpose of observing what is important, how the elements are related, what are the causes and the effects, and what are the underlying reasons (Montaku, Kaittikomol, & Thiranathanakul, 2012; Robbins, 2011). Furthermore, (Sternberg & Sternberg, 2012) asserts that "teaching for analytical thinking means encouraging students to analyze, critique, judge, compare and contrast, evaluate, and assess". Analytical thinking is a complicated higher-order thinking and it is one of three attributes of talents (Sternberg, 1997). Those three attributes of talents are analytical thinking skill, synthesizing skill, and problem-solving skill which are essential skills to learn and to do daily activities. If a person can think analytically, the person is able to predict, plan, decide, and foresee what may happen in the future. It can be concluded that those included in analytical thinking are the ability to analyze, compare, evaluate, predict, criticize, and categorize elements.

Many researchers have formulated analytical thinking from multiple perspectives. Based on the perspective on its use, analytical thinking is a "model" of thinking which is used to organize information to be articulated (Arzarello et al., 2005). Based on the perspective of mathematical thinking domain, (Thomley & Greenwald, 2012) says that analytical thinking is a sub-domain of mathematical thinking which is equivalent to other higher-order thinking such as pattern recognition, generalization, abstraction, problem-solving, and mathematical proofing.

Based on its characteristics, (Parta, 2016) categorizes analytical thinking to four parts. They are pre-analytical, partial analytical, semi-analytical, and analytical. Kinard and Kozulin (in (Parta, 2016) state that a person is said to be pre-analytical if the person only considers surface features of the task or problem and tend to apply the standard algorithm even it is not absolutely suitable to the task or problem given. Partial analytical thinking is indicated by parts of problem-solving

which are not logically connected though some parts of the solutions are analytical. Semi-analytical thinking is characterized by disturbing “elements” which lead to the breaking of “logical” structure of problem-solving (Parta, 2016).

(Ruseffendi, 1991) gives some indicators of analytical thinking. They are 1) the ability to give reasons why an answer or an approach to a problem is reasonable, 2) the ability to make and evaluate a general conclusion based on investigation or research, 3) the ability to predict or elaborate a conclusion or a decision from appropriate information, 4) the ability to validate arguments with deductive or inductive thinking, and 5) the ability to use supporting data to explain why the method used in solving the problem is correct. Anwar & Mumthas, (2014) state that analytical thinking consists of six steps. They are 1) identifying problems; 2) providing sources; 3) presenting and classifying information; 4) formulating strategies; 5) monitoring problem-solving strategies; and 6) evaluating solutions. The steps of analytical thinking are essentially starting from a problem which will be solved through logical, reasonable, and systematic steps. Also, it should be based on evidence and undergone a verification process so that the solution is accountable.

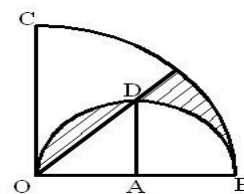
Analytical thinking cannot be observed directly because the nature of analytical thinking is abstract. Therefore, analytical thinking should be assessed indirectly through observing the behavior and response in solving a problem. Parta(2016) asserts that analytical thinking is used to solve non-routine problems. According to Polya (in Orton, 2004), solving a problem needs four steps. They are 1) understanding the problem, 2) devising a plan, 3) carrying out the plan, and 4) looking back. Musser, Burger, & Peterson (2011) view that Polya’s steps in solving a problem can be elaborated into several points. In the step of understanding the problem, someone encounters some questions such as (a) is the available information sufficient? (b) is there any secondary information? In the step of devising a plan, someone is faced to questions that deal with selecting appropriate strategies to solve the problem. After undergoing those two steps, someone can carry out the chosen strategy until the problem is solved or a new action is recommended. Thus, analytical thinking is indeed a part of problem-solving because it includes the ability to analyze, compare, evaluate, predict,

criticize, and categorize elements. Problems related to polygon areas can be a main choice to know the students' analytical thinking.

The concepts of the area of polygons have a broad application in education, science and technology, as well as in everyday life (Kow & Yeo, 2008; Mulligan, Prescott, Mithcelmore, & Outrhed, 2005) Calculating how many tiles needed for a certain area of floor, calculating the distance of an object based on the area of a graph on uniform rectilinear motion, calculating people density, and calculating the magnitude of pressure in physics are examples of its application. From its broad application, students are expected to have a good understanding of the concept at early age. The concept of the area of polygons is first studied at elementary schools, so the students' understanding at this level will have an impact on their success in studying advanced materials related to the concept. Elementary school students are expected to have comprehensive understanding of the concept of the area of polygons.

However, in fact, there are still many elementary school students who do not possess such understanding. In some initial observations, it is indicated that the students did not apply analytical thinking in solving the problems of polygon areas. Therefore, it can be said that the students have not yet understood the concept comprehensively. There is a strong indication that the students understand only the procedural concept of the area of polygons. It means that the students are able to calculate the area of polygons which is studied regularly at schools such as square, rectangle, parallelogram, circle, and the like. The students were not able to see the relations among the polygons which had been learnt previously in the given problems to calculate the areas of unfamiliar polygons. This can be seen in the excerpts of the elementary school students' work in solving the area of following polygons.

The quadrant in this picture has 14 cm of radius and centered at O . Point A is a midpoint of OB which is a center of the semicircle and it is through ODB . Line AD is perpendicular to line



OB. Find out the shaded area?

An excerpt of students' work is presented below.

$r_1 = 14$ $L \frac{1}{4} \text{ lingkaran} = \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$ $= 154$ $r_2 = 7$ $L \frac{1}{2} \text{ lingkaran} = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$ $= 77$ $L \text{ arsir} = L \frac{1}{4} \text{ lingkaran} - L \frac{1}{2} \text{ lingkaran}$ $= 154 - 77$ $= 77$ <p style="text-align: center;">(a)</p>	<p style="text-align: center;">A</p> $L \frac{1}{4} \text{ lingkaran} = \frac{1}{4} \times \frac{22}{7} \times 14 \times 14 = \frac{616}{2} = 308$ <p style="text-align: center;">B</p> $L \frac{1}{2} \text{ lingkaran} = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 77$ <p style="text-align: center;">(b)</p>
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Figure 1. Students' Work

In excerpt (a), the student did the calculation algorithmically (procedurally) without paying attention to the main question. Then, the student determined the shaded area by subtracting the area of semicircle centered at A from the area of quadrant centered at O. Same thing happened in excerpt (b), the student calculated the area of quadrant centered at O and calculated the area of semicircle centered at A then found no solution. Based on these preliminary findings, the study of analytical thinking of elementary school students in solving of polygon areas is important.

METHOD

This study is a qualitative study with descriptive exploratory design. The subjects of this study are taken from students of a mathematics olympiad team at elementary school level. The problems given are about polygon areas which are developed from the mathematics Olympiad competition for elementary school students at the provincial level in 2007. The problems have been presented in Figure 1 above. Data collection is done by giving a problem to the subject of the

study and asking them to solve it individually. Analytical thinking in solving the problem of polygon areas can be seen from students' analytical written answers and confirmed by the results of students' interview. Diagram 1 below shows the expected analytical thinking structure.

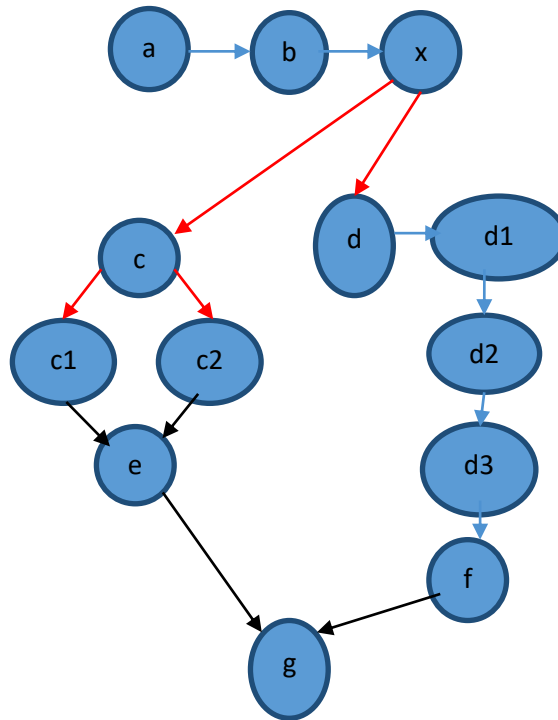


Diagram 1. The Structure of Analytical Thinking

Co	Description
de	
a	Identifying the problem
b	Determining a strategy to solve the problem
x	Focusing to shaded area
c	Determining the area through point <i>O</i> and <i>D</i>




Code	Description
c1	Calculating the area of the quadrant centered at A
c2	Calculating the area of ΔOAD
d	Determining the area through point B and D
d1	Determining $\angle DOA$
d2	Determining the area of the octant centered at O
d3	Subtracting the area of ΔOAD and the area of the quadrant centered at A from the octant centered at O
e	Obtaining the Area I
f	Obtaining the area IV
g	The solution
	Next step
	Integration
	Analysis

Table 1. The Descriptions of Thinking Structure Diagram

RESULT AND DISCUSSION

Result

The subject analytical thinking appears from the beginning when the subject identifies the problems then breaks it down into four parts, i.e. Part I, Part II, Part III and Part IV. Then, the subject solves the problem step by step as illustrated in Figure 2 below.

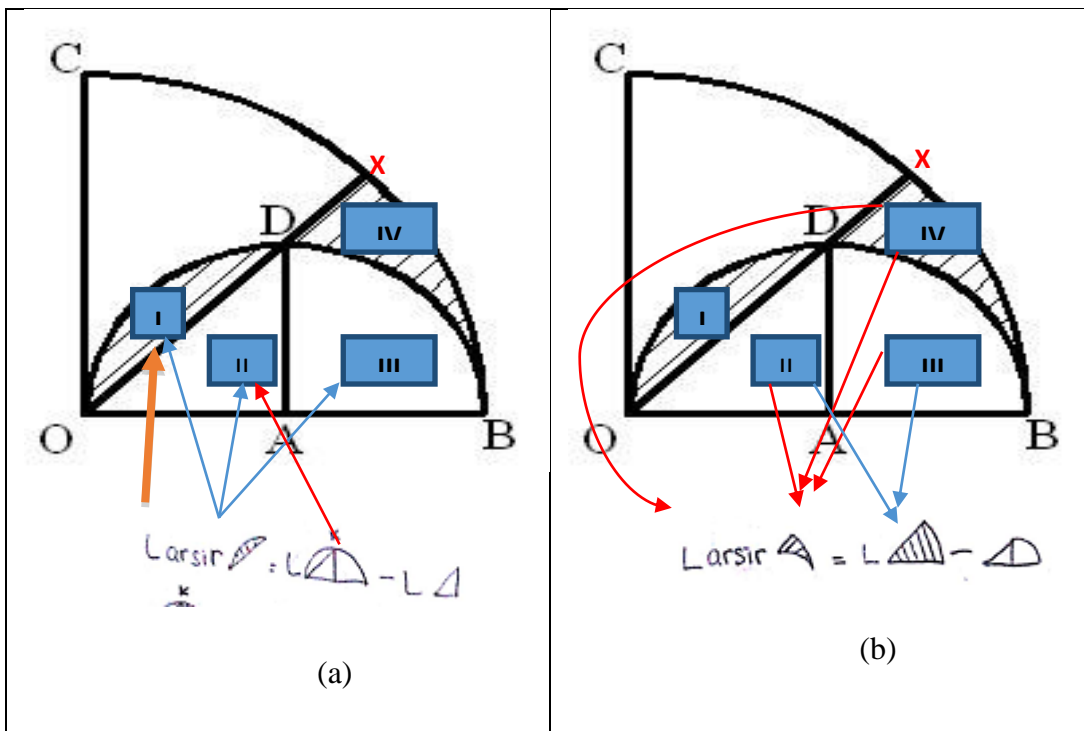


Figure 2. Illustration

Picture (a) shows the subject's plan in finding the area I by determining the area of semicircle centered at A or the areas of I + II + III minus the area of $\triangle OAD$. In picture (b), the subject's plan to determine the area IV by calculating the areas II + III + IV minus the areas II + III.

Next, the subject carries out the plan illustrated in picture (a) by determining the area I starting with giving the code k which means small circle with $r = 7$, i.e. semicircle centered at A as illustrated in Figure 3 below.

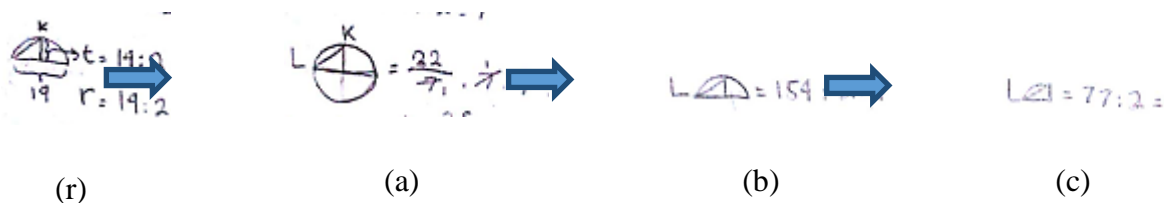


Figure 3. Chain of Implications

Figure 3 is a chain of implications $r \rightarrow a \rightarrow b \rightarrow c$ which ends with obtaining the area of the small quadrant centered at A. First, in picture (r) above, the subject states that the radius of a circle centered at A is $r = 14 \div 2 = 7$ and $= 14 \div 2 = 7$ with an intention of the height of $\triangle OAD$ is also 7. Next, in picture 3(a), the subject calculates the area of a circle centered at A which is $\frac{22}{7} \cdot 7 \cdot 7 =$

154. Then, in picture 3(b) $154 \div 2 = 77$ is the area of the semicircle centered at A and picture 3(c) $77 \div 2 = 38,5$ is the area of the quadrant centered at A . After that, the subject calculates the area of ΔOAD and integrates it into step in picture 3(c) to obtain the area I as shown below.

$$L_{arsir} = 38,5 - 24,5 = 14$$

Figure 4. The Area I

Thus, the area I is equal to the area of the quadrant centered at A minus the area ΔOAD and the subject obtains $L I = 38,5 - 24,5 = 14$.

To find out the area IV, first the subject explains that ΔAOD is an isosceles right triangle because $AO = AD$ and $\angle DOA = 90^\circ$. The subject declares that $\angle DOA = 90^\circ$ because the subject have understood that two perpendicular lines form right angle and the right angle is 90° . Also, the subject declares that $AO = AD$ because the radius of a circle has equal length. This indicates that the subject understands the principle applied in that problem. Then, the subject states that the sum of degrees in the triangle is 90° and exemplifies $\angle DOA = \angle x$. After that, the subject declares that $\angle DOA = \angle AOD$ because ΔAOD is an isosceles triangle which its vertex angles are equal. Thus, $\angle x + \angle x = 2 \angle x = 180^\circ - 90^\circ = 90^\circ$. Eventually, the subject obtains $\angle DOA = \angle x = 90^\circ \div 2 = 45^\circ$. Subject's step by step solving in determining $\angle DOA$ is shown below.

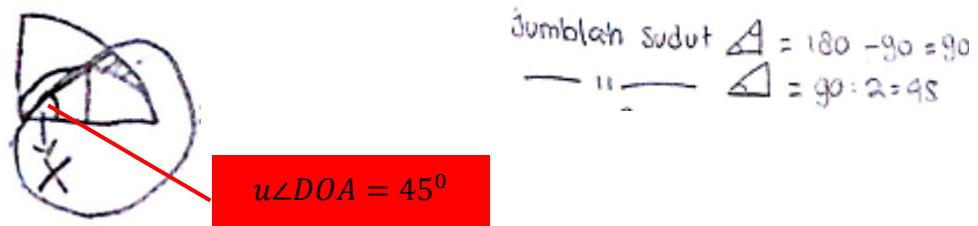


Figure 5. $\angle DOA$

Next, the subject determines the area of sector OBX by dividing $\angle DOA$ with 360° and then multiplying it by the area of circle centered at O . It is found the the area of sector $OBX = 77$. Prior to this step, the subject gives code B above the circle which means big circle with $r = 14$. Figure 6 below illustrates the steps taken in determining the area of sector OBX .

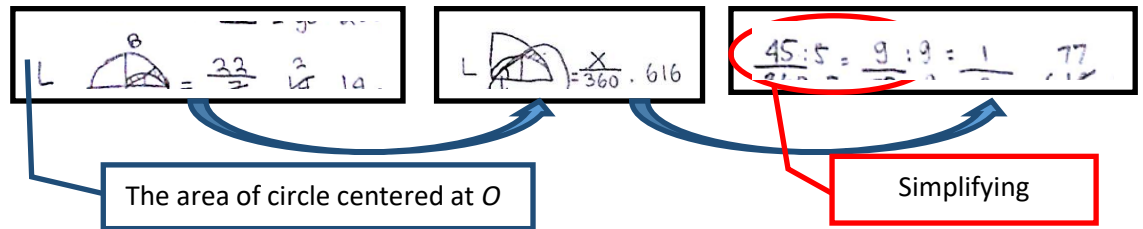


Figure 6. Sector OBX

After obtaining the area of sector OBX , the subject then determines the area IV which is the areas II + III + IV minus the areas II + III. It is found that $L.IV = 77 - 63 = 14$ as shown below.

$$L_{\text{arsir}} = L_{\text{shaded}} - L_{\text{triangle}} = 77 - 63 = 14$$

Figure 7. The area IV

Lastly, the subject integrates “key” information to obtain a solution from the problem, i.e. the area I added to the area IV as displayed below.

$$\begin{aligned} L_{\text{arsir}} &= 19 + 19 \\ &= 38 \end{aligned}$$

Figure 8. The shaded area

All steps taken by the subjects show that the subjects understand the problem given, therefore the subjects are able to find a solution with logical steps based on valid argumentation. This situation is in line with the character of analytical thinking proposed by Kinard and Kozulin (in Parta, 2016) who say that the subjects find the core applied to solve the problems. Besides, when we look the steps thoroughly, the steps are effective. It means that to reach the conclusion, the subjects do not do redundant or unnecessary steps.

In this study, the researcher found that the students have semi-analytical thinking. The semi-analytical thinking of the students is indicated by the presence of the so-called “disturbing elements” which broke “logical” structure of problem-solving (Parta, 2016). Figure 9 below shows the “disturbing elements”.

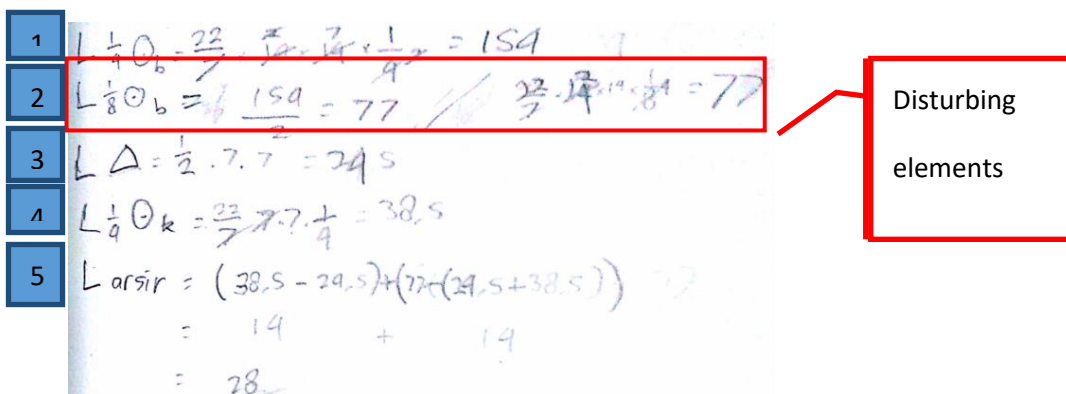


Figure 9. Semi-analytical thinking

The subject's solution is right, but actually there is an illogical step taken by the subject. This fact is revealed during the interview with the student. The subject states that the area of sector OBX is equal to one eighth multiplied by the area of the circle centered at O , however the subject is unable to give the right reason for the argument. The subject declares that the area of sector OBX is equal to one eighth multiplied by the area of the circle centered at O . This is based on assumptions and it is not based on premises which support the logical conclusions. According to Cockburn (2005:9), this subject is said to make implication errors, i.e. paying less or no attention to crucial thing of the problem. The crucial thing here is $\angle DOA$ which is the "key" information to obtain the area of sector OBX . Step 1, 3, and 4 in Figure 9 are valid, but the disturbing element is in Step 2 which resulted to the breaking of logical structure of problem solving done by the subject. The semi-analytical thinking of the subject can be seen in the Diagram 2 below.

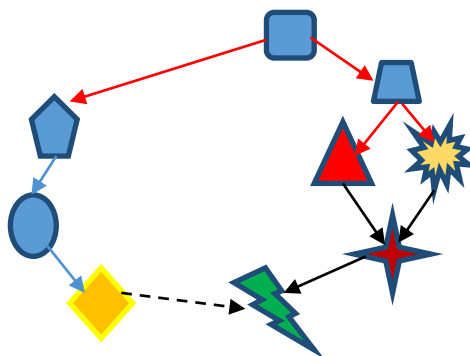


Diagram 2. The illustration of semi-analytical thinking














Symbol	Description
	The area of ΔOAD
	The area of the quadrant centered at A
	The area passing through point O and D
	Solution
	Disturbing element
	The area of circle centered at O
	Next step
	The breaking of the chain of implications
	Integration
	Analysis
	The problem
	Determining the area passing through point O and D
	Determining the area passing through point B and D

Table 2. Description of Semi-analytical thinking

Discussion

(Zhang, 2005) says that “analytic thought is defined as detaching the object from its context, a tendency to focus on attributes of the object, to assign it to categories, and a preference for using rules about the categories to explain and predict the object’s behavior”. From Zhang’s definition, the semi-analytical thinking is caused by the determination of categories on the elements attached to the problem. This is seen from the subject’s reason about the area of OBX . The subject merely assumes that the sector is equal to the octant. The subject arrives to that conclusion because the subject has experienced similar thing so that the subject assumes that the step is right.

Vinner, (1997) calls this kind of thinking as pseudo-analytical thinking. The main characteristic of the pseudo-analytical thinking is the absence of the procedure of control or reflection. The subject responds spontaneously without realizing what the subject does to solve the problem. When the subject solves the problem, the subject has no intention to control or verify the solution. In this case, the subject applied superficial similarities which are similarities in the shallow problem (Subanji, 2011). This situation, of course, disadvantage students. Hence, further study is required to scrutinize the pseudo-analytical thinking especially from the beginning when the error in the thinking process emerges.

CONCLUSION

The subjects' analytical thinking in solving polygon areas is shown by several indicators. They are algorithm clarity, chronological reasoning, valid argumentation, and effective steps taken. The subjects' answers show gradual problem solving. The algorithm clarity means that every step taken shows clearly the information being searched. Chronological reasoning means that there is a logical relationship among the steps. Effective steps mean that there are no unnecessary steps to reach to the conclusion. Semi-analytical thinking is characterized by the presence of "disturbing elements" which break the chain of implications in solving the problem. Those "disturbing elements" emerge because the subject utilized invalid information in solving the problem.

In addition, the subjects of this study are elementary school students who are expected to have comprehensive understanding of the concept of polygon areas. The subjects' understanding of this concept is important in studying advanced materials. Besides, the application of the concept of polygon areas is broad. Appropriate teaching method tailored to the students' thinking possibilities so that knowledge and learning experiences are well internalized is needed.

REFERENCES

- Anwar, B., & Mumthas, N. S. (2014). Taking Triarchic Teaching To Classrooms : Giving Everybody A Fair Chance. *International Journal of Advanced Research*, 2(5), 455–458.

- Arzarello, F., Ferrara, F., Robutti, O., Paola, D., Matematica, D., & Torino, U. (2005). The genesis of signs by gestures . The case of gustavo. *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*, 2, 73–80.
- Dunn, R., Honigsfeld, A., Doolan, L. S., Bostrom, L., Russo, K., Marjorie, S., Tenedero, H. (2010). Impact of Learning-Style Instructional Strategies on Students ' Achievement and Attitudes : Perceptions of Educators in Diverse Institutions. *A Journal of Educational Strategies, Issues and Ideas*, (June 2013), 37–41. <https://doi.org/10.3200/TCHS.82.3.135-140>
- Greene, B. A., Miller, R. B., Crowson, H. M., Duke, B. L., & Akey, K. L. (2004). Predicting high school students cognitive engagement and achievement : Contributions of classroom perceptions and motivation. *Contemporary Educational Psychology*, 29, 462–482. <https://doi.org/10.1016/j.cedpsych.2004.01.006>
- Kow, K., & Yeo, J. (2008). Teaching Area and Perimeter : Mathematics-Pedagogical-Content. *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia M. Goos, R. Brown, & K. Makar (Eds.)*, 621–627.
- Kuhn, J., & Holling, H. (2009). Exploring the nature of divergent thinking : A multilevel analysis. *Thinking Skills and Creativity*, 4, 116–123. <https://doi.org/10.1016/j.tsc.2009.06.004>
- Lopez, J. E., & Tancinco, N. P. (2016). Students Analytical Thinking Skills And Teachers ' Instructional Practices In Algebra In Selected State Universities And Colleges. *International Journal Of Engineering Sciences & Research Technology*, 3.00(6), 681–697.
- Montaku, S., Kaittikomol, P., & Thiranathanakul, P. (2012). The Model of Analytical Thinking Skill Training Process. *Research Journal of Applied Science*, 7, 17–20.

- Mulligan, J., Prescott, A., Mithcelmore, M., & Outrhed, L. (2005). Taking a Closer Look at Students' Images of Area Measurement. *APMC*.
- Musser, G. L., Burger, W. F., & Peterson, B. E. (2011). *Mathematics For Elementary Teachers A Contemporary Approach* (Ninth Edit). United States of America: John Wiley & Sons, Inc.
- Orton, A. (2004). *Learning Mathematics Third Edition* (third). London: Continuum.
- Parta, I. N. Karakteristik Berpikir Analitis Mahasiswa dalam Menyelesaikan “Masalah Sederhana” (2016). <https://doi.org/10.13140/RG.2.1.4728.2167>
- Robbins, J. K. (2011). Problem Solving, Reasoning, and Analytical Thinking in a Classroom Environment. *The Behavior Analyst Today*, 12(1), 40–47.
- Ruseffendi, E. T. (1991). *Pengantar Kepada Membantu Guru Mengembangkan Kompetensinya dalam Pengajaran Matematika untuk Meningkatkan CBSA*. Bandung: Tarsito.
- Sternberg, R. . (1997). *A thiarchie view of giftedness: theory and peactice, in: N. Colanggelo and G. A. Davis. Handbook of gifted education* (Second Edi). USA: Allyn and Bacon.
- Sternberg, R. J., & Sternberg, K. (2012). *Cognitive Psychology* (Sixth Edit). USA: Wadswort.
- Subanji. (2011). Teori Berpikir Pseudo Penalaran Kovariasional. In *Teori Berpikir Pseudo Penalaran Kovariasional*. Malang: UM Press.
- Sudibyoy, E., Jatmiko, B., & Widodo, W. (2016). The Effectiveness of CBL Model to Improve Analytical Thinking Skills the Students of Sport Science. *International Education Studies*, 9(4), 195–203. <https://doi.org/10.5539/ies.v9n4p195>
- Taleb, H. M. (2016). Enhancing Student Critical And Analytical Thinking Skills At A Higher Education Level In Developing Countries : Case Study Of

The British University In Dubai. *Journal Of Educational And Instructional Studies In The World*, (February), 67–77.

Thaneerananon, T., Wannapong, T., & Nokkaew, A. (2016). Development of a Test to Evaluate Students ' Analytical Thinking Based on Fact versus Opinion Differentiation. *International Journal of Instruction*, 9(2).
<https://doi.org/10.12973/iji.2016.929a>

Thomley, J. E., & Greenwald, S. J. (2012). *Encyclopedia of Mathematics and Society*. Salem Press.

Vinner, S. (1997). The pseudo-conceptual and the pseudo-analytical thought processes in mathematics learning. *Educational Studies in Mathematics*, 34, 97–129.

Windsor, W. J. J. (2008). Algebraic Thinking- More to Do with Why , Than X and Y.

Zhang, L. (2005). Does teaching for a balanced use of thinking styles enhance students achievement ? *Personality and Individual Differences*, 38, 1135–1147. <https://doi.org/10.1016/j.paid.2004.07.011>