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External Didactic Transposition of the Definite Integral Concept

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Abstract. *Calculus is a course included in the undergraduate curriculum of most science and engineering departments, including mathematics education. The definite integral is a fundamental concept in calculus, formally defined as the limit of a Riemann sum. However, in teaching practice, this concept is often reduced to a computational procedure using the fundamental theorem of calculus without emphasizing the meaning of the limit and the approximation process. This study aims to analyze the external didactic transposition of the concept of the definite integral as the limit of a Riemann sum, specifically, and to examine the alignment of Scholarly Knowledge and Knowledge to be Taught based on textbooks and curriculum documents. The research method used is a qualitative descriptive approach. This type of research is descriptive. The results of the study indicate that there is consistency between Scholarly Knowledge and Knowledge to be Taught, as reflected in textbooks and semester learning plan (RPS) documents in the curriculum, regarding the concept of definite integrals. However, given the time allotted for the learning process, simplifications may occur, potentially reducing students' learning experiences.*

Keywords: *external didactic transposition, definite integral, Riemann sum*

INTRODUCTION

Calculus is one of the courses included in the undergraduate curriculum for most science and engineering majors, including mathematics education. Calculus is viewed as an introductory course in preparing for advanced mathematics (Zollman, 2014; Oktaviyanthi et al., 2018). Therefore, students must possess an accurate understanding of the core mathematical concepts in this course, such as limits, derivatives, and integrals (Mokhtar et al., 2013).

The integral is one of the fundamental concepts in calculus and plays a crucial role. (Nurhayati, 2025) This concept serves as the foundation for various academic disciplines and contributes to the development of modern technology. Integrals play a vital role in various fields of knowledge, including the development of human cognitive abilities. Students should master the concept of integrals so that they can apply it to solve real-world problems (Johnson et al., 2022).

Although integrals are quite important for students, learning them is not easy. Many challenges arise in the study of integrals. The results found of Setyawan & Astuti (2021) indicated that integrals are a topic students find difficult to understand. Generally, students struggle with calculating integrals, particularly with distinguishing between variables and constants during integration (Nguyen & Rebello, 2011). Similarly, Fadillah (2019) and Monariska (2019) state that students face difficulties in learning integrals. Another issue that arises is the lack of understanding of integral concepts among civil engineering students, as revealed by Nurhayati et al. (2023a, 2023b). Another issue highlighted by Awaludin et al. (2020) was that challenges in integral learning include low mastery of basic integral concepts and students' difficulty in creating models using integrals for problem-solving. Understanding of integral concepts is formed empirically through students' learning experiences. According to Prihandhika et al. (2020), experiential learning is processed and analyzed through reasoning, which ultimately leads to concepts that are typically applied through justification, a process of evidence-based truth-seeking.

The difficulties encountered in understanding the concept of an integral in general also apply to the concept of *a definite integral*. Mathematically, a definite integral is not only understood as a tool for calculating the area of a region, but also as a representation of the limit of an infinite summation through the Riemann sum.

However, in college-level courses, definite integrals are often presented directly as fundamental theorems of calculus and rules for calculating integrals. This approach leads students to view definite integrals as a mechanical procedure: namely, substituting upper and lower limits into an antiderivative. Research shows that students have difficulty interpreting definite integrals as a process of limits and accumulation (Orton, 1983). Studies conducted by Sealey (2012) and Hanafi (2021) also indicate errors in calculating definite integrals using Riemann sums

Several studies indicate that students have difficulty understanding definite integrals conceptually. Orton (1983) and Thompson (1994) assert that students tend to view definite integrals as algebraic procedures rather than as the limit of a summation process. This difficulty becomes even more apparent when students are asked to relate definite integrals to geometric interpretations or the concept of accumulation.

Further research highlights students' weak understanding of the Riemann sum as the foundation of the definite integral. Tall and Vinner (1981) demonstrated a conflict between *the conceptual image* and *the conceptual definition* of the integral, in which students recognized the integral formula without understanding the underlying limit process. Similar results were also reported by Sealey (2006), who found that students struggled to interpret definite integrals as approximations of area via interval partitioning.

A study conducted by Utari and Utami (2020) found that students tend to apply basic calculus theorems directly without understanding integration as a process of accumulation. However, these studies generally analyze students' errors from cognitive and procedural perspectives, without tracing their sources from a didactic perspective. Therefore, a study of didactic transposition of the concept of the integral, specifically through the calculation of the Riemann sum, remains open to in-depth investigation.

According to Brousseau (2002) and Chevallard (1989), didactic transposition is the transfer of knowledge from the created and applied to the knowledge to be taught, as shown in Figure 1.

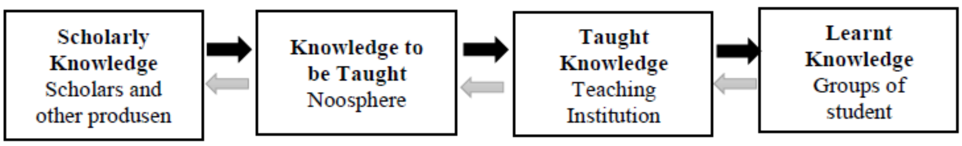

Figure 1. The Didactic Transposition Process

The didactic transposition process is carried out through external transposition and internal transposition, as shown in Figure 2. External didactic transposition refers to the shift from scientific to educational mathematics. This external didactic transposition involves transforming, interpreting, and re-elaborating scientific knowledge into instructional content. As a result of this re-elaboration, not all scientific mathematics is included in the school curriculum, leading to differences between scientific literature and didactic texts with their specific instructional tasks. Curriculum designers and authors of mathematics textbooks for elementary and secondary schools carry out this process. Additionally, the transfer of school mathematics content into classroom teaching within the context of the teaching-learning process is carried out by instructors as part of the process known as internal didactic transposition (Paun, 2006).

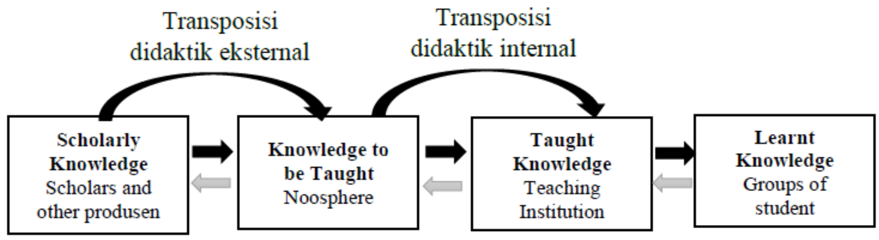

Figure 2. Internal and External Factors in the Transposition Process

Thus, this study aims to analyze the external didactic transposition of the concept of definite integrals as Riemann sums in particular and to examine the alignment of *Scholarly Knowledge* and *Knowledge to be Taught* based on the textbooks and curriculum documents under review. The benefit of analyzing the external didactic transposition of the concept of definite integrals is to identify conceptual distortions. Furthermore, presenting the concept in an overly formal manner from the outset can increase the complexity of information processing and hinder conceptual understanding. With external didactic transposition, visual representations, contextual examples, and heuristic approaches can serve as a bridge toward higher-level abstraction (Artigue, 1994). Furthermore, research on the external didactic transposition of the concept of definite integrals has not been extensively conducted; thus, this study will provide new insights into this topic.

RESEARCH METHOD

This study employs a qualitative descriptive approach. This type of research is descriptive. Descriptive research aims to describe the process of didactic transposition in definite integral material. Analysis is a qualitative data collection technique involving the collection and analysis of documents related to the research subject (Bowen, 2009). This analysis was conducted on the first two parts of didactic transposition: Scholarly Knowledge and Knowledge to be Taught. This study analyzed two data sources, namely university textbooks and curriculum documents. The university textbook was selected from the definite integral material found in Calculus and Analytic Geometry, Volume 1, Fifth Edition (1987) by Edwin J. Purcell and Dale Varberg, which is used as one of the references in the Calculus course for a specific program of study in Maluku Province, serving as Scholarly Knowledge.

Meanwhile, the curriculum documents were obtained from the program's website and were used to examine the Knowledge To Be Taught regarding definite integrals. The analysis in this study focuses on the alignment between Scholarly Knowledge and Knowledge to be Taught regarding the concept of definite integrals. The study of the definite integral concept as Scholarly Knowledge, conducted in the textbook Calculus and Analytic Geometry, Volume 1, Fifth Edition, 1987, by Edwin J. Purcell and Dale Varberg, begins by examining the definition of the Riemann sum, the process of performing the Riemann sum, and concludes with the definition of the definite integral as the limit of the Riemann sum. Subsequently, a study of definite integrals as Knowledge to be Taught was conducted by examining documents integral to the curriculum, namely the course distribution, course descriptions, and course syllabi containing the concept of definite integrals, followed by a descriptive analysis related to the didactic process of transposition.

RESULTS AND DISCUSSION

The examination of definite integral concepts as Scholarly Knowledge or scientific knowledge is conducted through calculus textbooks, while Knowledge to be Taught or knowledge to be taught is examined through curriculum documents. This is intended to illustrate the status of definite integral concepts as depicted in both documents.

Definite Integral Content in Textbooks as Scholarly Knowledge

The textbook widely used in higher education, which serves as a primary reference informing the curriculum of this study program—particularly in the Calculus course—is the translated Edition, Calculus and Analytic Geometry, Volume 1, Fifth Edition (1987), by Edwin J. Purcell and Dale Varberg. In this textbook, Chapter Five is devoted specifically to the topic of integrals, encompassing several subtopics, namely: antiderivatives (indefinite integrals), an introduction to differential equations, summation and sigma notation, an introduction to area, definite integrals, the Fundamental Theorem of Calculus, further properties of definite integrals, as well as techniques supporting the computation of definite integrals. Accordingly, the concept of the definite integral is introduced only after students have engaged with a set of prerequisite topics that underpin its conceptual understanding.

In this article, we will discuss the concept of definite integrals, starting with the topic of Riemann sums.

Riemann Sum. Consider a function f defined on the closed interval $[a, b]$. This function may take positive or negative values, and it does not even need to be continuous. Its graph might look like Figure 3.

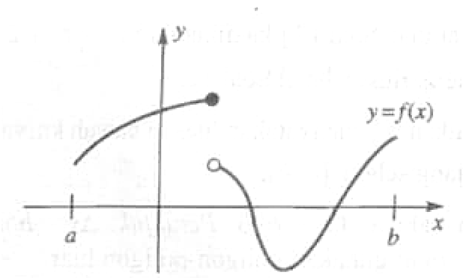

Figure 3. The function f defined on the closed interval $[a, b]$

Consider a partition P of the interval $[a, b]$ into n subintervals (which need not be of equal length) using the points $= x_0 < x_1 < x_2, \dots < x_{n-1} < x_n = b$, and let $\Delta x_i = x_i - x_{i-1}$. In each subinterval $[x_{i-1}, x_i]$, take an arbitrary point $\overline{x_i}$ (which may be an endpoint); we call this point as the sample point for the *i*-th interval. For example, in Figure 4, this is constructed for $n = 6$.

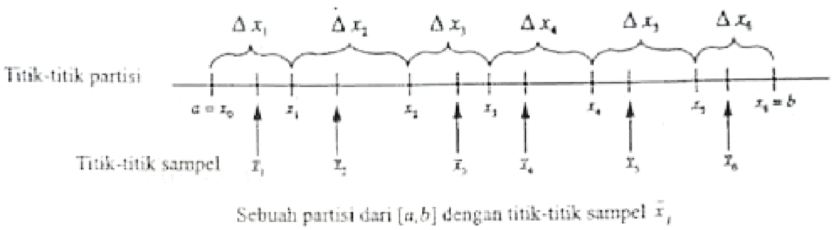

Figure 4. Partition of a closed interval $[a, b]$

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Next, the following sum is formed

$$R_p = \sum_{i=0}^{n} f(\bar{x}_i)\,\Delta x_i$$

We denote R_p as the **Riemann sum** corresponding to the partition P, the geometric interpretation can be seen in Figure 5. It describe about the contribution of a rectangle along *the x-axis* as the negative of its area because in this case $f(\bar{x}_i) < 0$

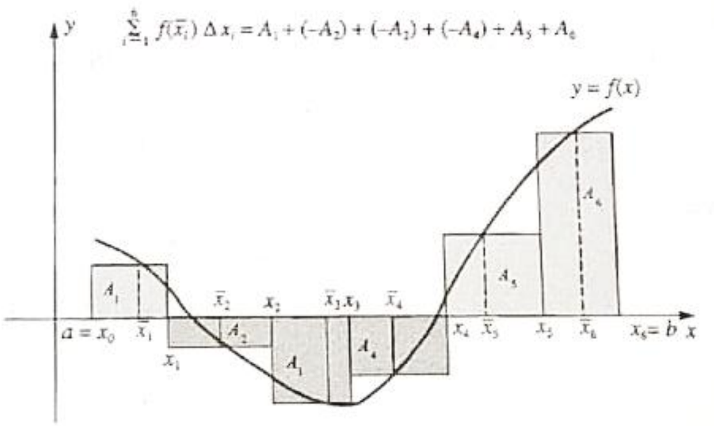

Figure 5. Geometric interpretation of the Riemann sum

Definition of a Definite Integral. Let *P*, Δx_i , and $\bar{x}_i$ have the meanings discussed above. Next, let$|P|$ be the norm of *P*, which represents the length of the longest interval in the partition of *P*. Then, the definition of a definite integral is as follows.

Definition:

Let *f* be a function defined on the closed interval [*a*,*b*]. If

$$\lim_{|P|\to 0} \sum_{i=0}^{n} f(\bar{x}_i)\,\Delta x_i$$

exists, then$f$ is said to be integrable on $[a, b]$. Furthermore,$\int_a^b f(x)dx$ is called the definite integral (or Riemann integral) of f from *a* to *b*, given by

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$$\int_a^b f(x)dx = \lim_{|P|\to 0} \sum_{i=0}^{n} f(\bar{x}_i)\,\Delta x_i$$

Given an integrable function, we can compute its integral using a regular partition (intervals of equal length) and convenient sample points (the endpoints of the intervals). Thus, if $|P| \to 0$, then the sum of the partitions or intervals is $n \to \infty$; therefore, the definite integral of f from a to b can be written as

$$\int_a^b f(x)dx = \lim_{n\to \infty} \sum_{i=0}^{n} f(\bar{x}_i)\,\Delta x_i$$

Definite Integral Material in the Study Program Curriculum as Knowledge to be Taught

This section examines three documents that are integral to the university mathematics program curriculum in Maluku, Indonesia. These documents were obtained from the program's website. The three documents are the course catalog, course descriptions, and RPS for the course on definite integrals.

In general, as outlined in the curriculum for this study program, 96 courses are offered. These consist of 42 required courses totaling 117 credit hours, with the remaining courses being electives, from which students must take a minimum of 27 credit hours to fulfill the 144-credit-hour minimum for the bachelor's degree. From the course descriptions included in the curriculum, definite integrals are certainly covered in the Calculus course, as shown in Figure 6. In the Calculus course description, the topics covered include antiderivatives, definite integrals, applications of definite integrals, transcendental functions, and integration techniques. Meanwhile, the topics covered in definite integrals include Riemann sums, theorems on definite integrals, and the fundamental theorem of calculus.

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15	Nama Mata Kuliah	:	**Kalkulus**
	Kode Mata Kuliah	:	MAT234315
	Jumlah SKS	:	3
	Deskripsi Mata Kuliah	:	Mata kuliah ini dimaksudkan untuk memberikan pengetahuan konsep-konsep matematika tentang Anti turunan (pengertian anti turunan, teorema-teorema, dan teknik anti turunan), Integral tentu (jumlah Riemann, teorema-teorema integral tentu, dan teorema dasar kalkulus), Aplikasi integral tentu (luas bidang, volume benda putar, luas permukaan benda putar, momen dan pusat massa), Fungsi Transenden (Fungsi logaritma dan fungsi eksponen), serta berbagai teknik pengintegralan (metode subitusi, integral trigonometri, metode subtitusi yang merasionalkan, integral parsial dan parsial berulang dan integral fungsi rasional).

Figure 6. Description of the Calculus Course

This Calculus course is offered in the second semester, as shown in Figure 7, and specifically as a prerequisite for students who have completed the Basic Mathematics course in the first semester. Thus, the Calculus course is a continuation of the Basic Mathematics course, covering real numbers, functions and their graphs, limits and continuity, derivatives, and their applications.

Kode MK	Nama Mata Kuliah	SKS	Prasyarat
MAT234112	Kewarganegaraan	2	
MAT234113	Bahasa Indonesia dan Teknik Penulisan Ilmiah	2	
MAT234114	Pengelolaan Sumber Daya Kelautan dan Pulau-Pulau Kecil	2	
MAT234315	Kalkulus	3	MAT234207
MAT234216	Pengantar Logika dan Himpunan	3	
MAT234217	Aljabar Linier Elementer	3	
MAT234218	Statistika Elementer	3(1)	
MAT234119	Program Linier	3(1)	
Total		**21**	

Figure 7. Distribution of Second Semester Courses

In a review of the course syllabus for Calculus prepared by the course instructor, there are differences in the topics covered in the definite integral section compared to those listed in the course description. In the syllabus, the topics presented in the definite integral material are more comprehensive, namely: the sigma notation, the mathematical induction, an introduction to area, the Riemann sum, definite integrals, and the Fundamental Theorem of Calculus, as shown in Figure 8.

3	Mampu memahami notasi sigma dan induksi matematika dalam menghitung luas menurut poligon luar dan poligon dalam	**Integral Tentu** ▪ Notasi Sigma ▪ Induksi Matematika ▪ Pendahuluan Luas	Metode Pembelajaran (pilih yang sesuai): ☐ PjBL ☐ CBL ☑ Ceramah ☑ Diskusi kelompok ☐ Simulasi ☐ Kolaboratif ☐ Koorporatif ☑ Tugas ☐ Lainnya	3 x 50′	✓ Mendengarkan penjelasan materi ✓ Memikirkan jawaban dari pertanyaan yang diberikan dosen ✓ Terlibat aktif dalam diskusi	Kemampuan menghitung luas daerah menurut poligon luar dan poligon dalam	8
4	Mampu menghitung integral tertentu dengan limit jumlah Riemann dan membuktikan beberapa teorema integral tertentu.	**Integral Tentu** ▪ Jumlah Riemann ▪ Defenisi Integral Tentu sebagai Jumlah Riemann	Metode Pembelajaran (pilih yang sesuai): ☐ PjBL ☐ CBL ☑ Ceramah ☑ Diskusi kelompok ☐ Simulasi ☐ Kolaboratif ☐ Koorporatif ☑ Tugas ☐ Lainnya	3 x 50′	✓ Mendengarkan penjelasan materi ✓ Memikirkan jawaban dari pertanyaan yang diberikan dosen Terlibat aktif dalam diskusi	Ketepatan dalam menghitung integral tentu menggunakan jumlah Riemann	8
5	Mampu menghitung integral tertentu berdasarkan teorema-teorema dasar kalkulus	**Integral Tentu** ▪ Teorema Dasar Kalkulus ▪ Metode Subtitusi Sederhana		3 x 50′	✓ Mendengarkan penjelasan materi ✓ Memikirkan jawaban dari pertanyaan yang diberikan dosen Terlibat aktif dalam diskusi	Ketepatan dalam menghitung integral tentu berdasarkan teorema dasar kalkulus	8

Figure 8. Course Syllabus for Calculus

Based on a study of definite integrals as scientific knowledge and as material to be taught, the course description document for calculus reveals a discrepancy between the material to be taught and that found in the scientific literature. However, in the syllabus document, the topics presented in the definite integral material are more comprehensive and align with the sequence of scientific knowledge. Nevertheless, a potential obstacle is the time allotted for the learning process. In the RPS, the time allocated to Riemann sums and definite integrals is considered very minimal, given the assessment criteria requiring accuracy in calculating definite integrals using Riemann sum limits. Consequently, during the learning process, the presentation of the concept of definite integrals may be simplified, leaving students without sufficient learning experience.

Another potential obstacle is the lack of continuity in the material to be taught regarding the concept of definite integrals using the Riemann sum approach. The Riemann sum is a limit at infinity, and this material is covered in the Basic Mathematics course offered in the previous semester. Another potential obstacle, as noted, is a difference in the instructors teaching the Basic Mathematics and Calculus courses.

The analysis conducted revealed several findings that can be explained in this study: the first finding shows that the formal definition of the definite integral as the limit of a Riemann sum tends to be omitted or treated as supplementary information in textbooks and curricula, as evidenced by the minimal time allocated to it in the curriculum guidelines. In formal mathematical literature, the definite integral is defined as the limit of partitions approaching zero. In contrast, in the analyzed curriculum documents, it is more often introduced as the "area under the curve," with little emphasis on the limit process. This indicates an ontological reduction: the object "limit of partitions," which serves as the conceptual foundation, is replaced by a visual-geometric representation. This reduction serves a pedagogical purpose but implies a disconnect in students' understanding of the meaning of approximation and the limit process as the essence of the integral. The second finding indicates a potential shift in epistemological orientation from a constructive to a procedural approach. While the integral, within a scientific framework, is defined as a limit of a Riemann sum, the presented curriculum focuses more on the fundamental theorem of calculus and substitution techniques. This shift leads to the definite integral being understood as a computational procedure rather than a conceptual process. This phenomenon reinforces the findings of previous studies, which indicate that calculus often undergoes "conceptual instrumentalization" (Bosch & Gascón, 2006). However, this research clarifies that such instrumentalization occurs at the stage of external transposition, not merely in classroom teaching practices. The third finding suggests a fragmented relationship between definite integrals and the fundamental theorem of calculus. Within the scientific framework, the two form a conceptual unity that fundamentally links derivatives and integrals. With a curriculum structure that separates derivative and integral material into separate courses across different semesters, this becomes a burden on the learning process. Thus, external didactic transposition offers strategic benefits for curriculum development and mathematics education, enabling the systematic, relevant, and developmentally appropriate adaptation of scientific knowledge to students' cognitive development without compromising its epistemological integrity.

CONCLUSION

Based on the discussion in the previous section, it can be concluded that the external didactic transposition of the concept of definite integrals in scientific studies, as outlined in textbooks and the curriculum, specifically the RPS, is appropriate in terms of the sequence of topics presented. However, given the time allotted for the learning process, simplifications may occur, potentially diminishing students' learning experience. A potential obstacle lies in the process of calculating definite integrals using Riemann sums, as there is no continuity of material within the calculus course itself. However, it is a continuation of the basic mathematics course from the previous semester. The findings of this study can serve as a reference for further research, with a focus on discussions with course instructors. Additionally, an in-depth study can be conducted on internal didactic transposition, specifically how textbook material is applied in the classroom learning process in accordance with the established curriculum. Hopefully, this study will contribute to improving curriculum documents and teaching materials.

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