
Case Study of Students' Misconceptions in Adding and Subtracting Fractions and Their Solutions

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Abstrak. Penelitian ini bertujuan untuk menganalisis penyebab terjadinya miskonsepsi siswa pada penjumlahan dan pengurangan pecahan berbeda penyebut, beserta solusinya berdasarkan diagram *fishbone*. Penelitian ini merupakan penelitian kualitatif dengan jenis studi kasus jamak. Subjek penelitian adalah siswa yang mengalami miskonsepsi pada penjumlahan dan pengurangan pecahan berbeda penyebut. Hasil penelitian ini yaitu, miskonsepsi siswa dalam penjumlahan dan pengurangan pecahan berbeda penyebut diklasifikasikan dalam enam aspek pada diagram *fishbone*: *man*, *machine*, *method*, *material*, *measurement*, dan *environment*. Hal yang menyebabkan kesalahan dalam penjumlahan dan pengurangan pecahan berbeda penyebut yaitu, siswa tidak mencari bentuk pecahan setara, tidak tersedianya media visualisasi untuk operasi pecahan berbeda penyebut, mobilitas guru yang terbatas dalam mengajar, dan suasana kelas yang tidak kondusif. Selain itu, bahasa yang digunakan dalam buku ajar tidak relevan dengan level kognitif siswa, dan adanya subjektivitas dalam penilaian jawaban siswa oleh guru. Tutor sebaya yang memberikan penjelasan yang salah kepada teman-temannya, adanya kelompok pertemanan tertentu, dan siswa dengan akademik tinggi yang memiliki kemampuan komunikasi rendah juga menjadi faktor penyebab miskonsepsi. Melalui pemahaman penyebab miskonsepsi tersebut dapat menjadi referensi bagi guru untuk mencegah miskonsepsi pada siswa.

Kata Kunci: miskonsepsi, pecahan, diagram *fishbone*

Abstract. *This research aims to analyze the causes of students' misconceptions in addition and subtraction with unlike denominators, along with their solutions, based on a fishbone diagram. This study was qualitative research with a collective case study design. The research subjects were students who experienced misconceptions in adding and subtracting fractions with unlike denominators. The result of this research was caused by student misconceptions in addition to the subtraction of fractions, unlike denominators, which are classified based on six aspects in the fishbone diagram: man, machine, method, material, measurement, and environment. This causes errors in calculation operations involving numerators and denominators without equalizing fractions, the absence of visual aids for fractions that are unlike denominators, limited teacher mobility in teaching, and a non-conducive classroom atmosphere. Moreover, the language used in textbooks may not be relevant to students' cognitive levels, and there may be subjectivity in teachers' assessments of student answers. Peer tutors who provide incorrect explanations to their friends, the existence of certain friendship groups, and high-category students with low communication abilities. Understanding the*

causes of these misconceptions can be a reference for teachers to prevent misconceptions in students.

Keywords: *misconceptions, fraction, fishbone diagram*

INTRODUCTION

The set of fractional number is $F = \left\{ \frac{a}{b} \mid a, b \in \text{non-negative integer}, b \neq 0 \right\}$ (Musser et al., 2011). The addition and subtraction fractions with unlike denominators theorem are shown in Figure 1.

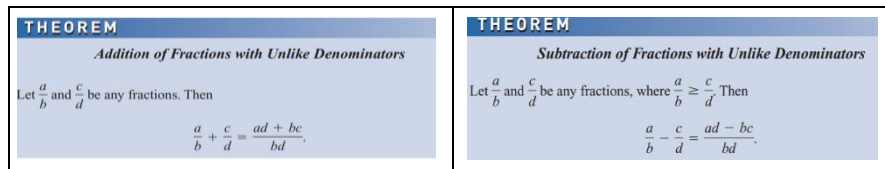


Figure 1. The Addition and Subtraction Fraction with Unlike Denominators Theorem

From Figure 1, in addition or subtraction of fractions with unlike denominators, such as $\frac{a}{b} + \frac{c}{d}$, The steps involved are as follows, (1) multiplying the denominators $b \times d$, (2) cross multiplication between the numerators and denominators $a \times d$ and $b \times c$, and (3) adding or subtracting the results of the multiplications located in the numerator $(a \times d) + (b \times c)$. Therefore, resulting $\frac{a}{b} + \frac{c}{d} = \frac{a(d)+b(c)}{b(d)}$. However, in subtracting fractions with unlike denominators, there is a specific condition $\frac{a}{b} \geq \frac{c}{d}$.

Misconceptions in adding and subtracting fractions with unlike denominators often occur among students (Rachmah, 2020). Researchers identified the indication of these misconceptions during the preliminary study through literature review in Table 1.

Table 1. Preliminary Study


Researcher	Research Title	Student Misconceptions
Fitri et al. (2018)	<i>Analisis Kesalahan Siswa dalam Menyelesaikan Soal Operasi Hitung Pecahan Kelas V Sekolah Dasar</i>	The students add numerators and denominators as follows: $\frac{3}{7} + \frac{5}{7} = \frac{8}{14}$
Efriani (2021)	<i>Penyelesaian Operasi Pecahan: Identifikasi Kesalahan Konsep</i>	The student subtracts numerator and denominator as follows: $\frac{5}{6} - \frac{4}{5} = \frac{1}{1}$

The misconception revealed in Table 1 is students directly add and subtract numerators and denominators without finding equivalent fractions. This study uses the methods of APKL and USG to analyze the discussed topic. First, using the APKL method where the topic raised must be “*aktual*” (A), “*problematik*” (P), “*kekhayalakan*” (K), and “*layak*” (L). Actual means truly happened and is currently being discussed. Problematic means need to be resolved promptly. Feasible means it concerns many people's lives. Furthermore, appropriate means the topic aligns with the researchers’ authority. The topic of students’ misconceptions and subtraction with unlike denominators fulfills these APKL aspects.

Actual

From an actual perspective, it was found that elementary students from UM Laboratory School made mistakes in solving addition and subtraction problems with denominators that were unlike denominators, resulting in incorrect answers. The student errors are presented in Table 2.

Table 2. Student's Mistake

Question	Student Answer	Student's Answer Translation
$\frac{3}{2} + \frac{1}{4} - \frac{2}{6} = \dots$		$\frac{3}{2} + \frac{1}{4} - \frac{2}{6} = \frac{2}{0}$

The student is suspected to have misconceptions in Table 2. This indication is reinforced by the results of Interview 1.

Interview 1. Confirmation of Student Answers

- (A) Interviewer : “So according to you, the operation is numerator with numerator and denominator with denominator?”
 (B) Student : “Not sure, but something like that”
 (C) Interviewer : “ $\frac{3}{2} + \frac{1}{4} - \frac{2}{6} = \dots$ ”
 (D) Student : “ $3 + 1 = 4$, then $4 - 2 = 2$ ”
 (E) Interviewer : “So the denominator here is $2 + 4 - 6 = 0$, right?”
 (F) Student : “Yes”

Students proven to have misconceptions as shown in statements from line C to line F in Interview 1, when he add and subtract between numerators and

denominators. It causes students aren't aware with adding and subtracting fractions with different denominators theorem taught by the teacher or in the textbook.

Problematic

From a problematic aspect, this topic has a chain effect on the next level, specifically on the material of algebraic fraction operations in Grade VII. Based on research conducted by Aulia & Sutriyono (2018), students make mistakes in algebraic fraction operations in Figure 2.

1. b) Hasil penyelesaian dari $\frac{-6v-3z}{2v+z} + 4$ adalah?

$$\frac{-6v-3z}{2v+z} + \frac{-21v-12z}{8v+4z}$$

$$\frac{-30v-15z}{10v+5z} = \frac{-3v-3z}{1v+5z}$$

Figure 2. Student Error

Feasible

From a feasibility aspect, this reduction of fractions is applied in the calculation of inheritance division. In Islamic jurisprudence, if the heirs consist of both predetermined heirs (*dzawil furudh*) and undetermined heirs (*ashabah*) who may receive all or the remaining inheritance after it has been divided among the predetermined heirs, then this issue can be resolved by equating the numerator of their respective shares in fractional form by multiplying the numerator or finding the greatest common divisor of the numerators (Mandasari et al., 2022). For example, if the heirs consist of a daughter with a $\frac{1}{2}$ share, a husband with a $\frac{1}{4}$ share, and a father who is an undetermined heir as stated in Table 3.

Table 3. Division of Heirs of Dzawil Furudh and Ashabah

Heir	Furudh Muqaddarah	The main issue	Inheritance Assets
Daughter	$\frac{1}{2}$	4	2
Husband	$\frac{1}{4}$		1
Father	<i>ashobah</i>		1

(Mandasari et al., 2022)

The portion of wealth from *ashabah* calculated by subtracting the following fractions.

$$1 - \frac{1}{2} - \frac{1}{4} = \frac{4 - 2 - 1}{4} = \frac{1}{4}$$

Therefore, the portion of wealth from the heirs is $\frac{1}{4}$ of the inheritance.

Appropriate

From the appropriate aspect, researchers understand the concept of addition and subtraction of fractions through several approaches correctly. Researchers have also learned the concept of addition and subtraction of fractions in elementary school, junior high school, high school, and college in the Mathematics Education Undergraduate Program. Currently, the researchers are pursuing education in the Mathematics Education S1 Program in the sixth semester. During college, the researchers passed the Introduction to Algebra course, which included discussions about fractions. Therefore, the researchers can explain and analyze the issues being discussed.

After fulfilling APKL's indicators, researchers analyzed this topic using the USG method, where the topic addressed must meet urgency (U), seriousness (S), and growth (G). Urgency means, the availability of time to solve the problem is urgent. Seriousness means, if not addressed promptly, it will have particular impacts. Growth refers to how quickly negative growth is caused by the topic.

Urgency

From the urgency aspect, researchers interviewed with the elementary school mathematics teacher to determine the importance of this topic in Interview 2.

Interview 2. Confirmation of the Achievement of the Urgency Indicator on the Topic

(G) Researcher : "I found that students make a mistake by adding and subtracting between numerators and denominators, without finding equivalent fractions. In your opinion, how long should it take to solve such a problem?"

(H) Teacher : "Within one lesson"

(I) Researcher : "So within one day?"

(J) Teacher : "Yes, because one topic is completed in one day. If it's not finished, there will be a remedial session"

(K) Researcher : "How long is the remedial session?"

(L) Teacher : "According to the next math schedule"

The duration of time in resolving student's misconceptions was revealed in statements from lines G and H in Interview 2, which occurred during a single meeting. This is further supported by statements from lines I to L in Interview 2, indicating that one topic should be completed in one day. If there are students who haven't finished on that day, they are given additional exercises as remediation and collected in the next schedule.

Seriousness

From the seriousness aspect, if the solution to students' misconceptions isn't implemented in one meeting, students make similar mistakes in solving complex problems. These student errors were found by researchers at SDN 3 Dadapan, Wajak District, Malang Regency, in November 2023, as presented in Figure 3.

Dewi memiliki pita sepanjang $2\frac{1}{2}m$, pita tersebut diberikan kepada Ani sepanjang $\frac{2}{3}$ meter. Berapakah panjang pita Dewi sekarang?

$$2\frac{1}{2} - \frac{2}{3} = \frac{5}{2} - \frac{2}{3} = \frac{3}{1}$$

Figure 3. Student's Mistakes

In Figure 3, indicated that student didn't follow applicable procedure according the theorem, resulting incorrect answers in the story problem related to subtracting fractions with unlike denominators. The student's mistake was further confirmed through an interview conducted by the researchers with the elementary school mathematics teacher as presented in Interview 3.

Interview 3. Confirmation of Seriousness Indicator in Topic

(M) Researcher : "If not resolved in one learning session, could it have an impact on solving more complex problems, such as story problems related to the concept of addition and subtraction of fractions?"

(N) Teacher : "Certainly, the children's literacy is lacking. Besides, they always believe that mathematics is difficult and rarely do we find children who enjoy mathematics".

The indication of misconceptions results in errors in solving more complex problems, as validated in statements M and N of Interview 3. This is because such

misconceptions impact the solution of story problems related to specific concepts. Furthermore, the condition of students who consistently view mathematics as a daunting subject also contributes to their low motivation to learn. Therefore, this issue must be addressed promptly.

Growth

From a growth aspect, if students' misconceptions continuously lead to negative development. This indication is reinforced in Interview 4.

Interview 4. Confirmation of Growth Indicators in Topic

- (O) *Researcher* : "For example, if a student gives incorrect explanations to another student, does it result in misconceptions about the material they are taught?"
- (P) *Teacher* : "Yes, because academically low-achieving students usually just follow along"

The indication of negative growth is revealed in statements from lines O and P of Interview 4. If a student provides incorrect explanations to their friend, it can lead to similar misconceptions in the friend being taught. Furthermore, students in the low category often assume that concepts taught by high-category students are correct. Students in the low category tend to haven't motivation to understand their friend's explanations further, even though the concept taught by their friend may be incorrect.

The study related to students' misconceptions in addition and subtraction of fractions with unlike denominators through the APKL and USG methods validates that this topic is crucial to be comprehensively examined. This is because it can result in negative and fatal impacts, as stated in the APKL and USG analyses. The APKL analysis is presented in Table 4.

Table 4. APKL Analysis

Problem	Criteria				Information
	A	P	K	L	
Student Misconceptions in Adding and Subtracting Fractions	✓	✓	✓	✓	Qualify

The Teacher's Challenges in Creating Teaching Modules	✓	✓	-	✓	Not Qualify
The Issue of Using Two Languages in Mathematics Education in Integration Cambridge Curriculum and National Curriculum	✓	✓	✓	✓	Qualify
The Issue of Mathematics Learning Media	✓	✓	✓	✓	Qualify

From Table 4, some issues do not meet the criteria, resulting in their elimination in the USG analysis presented in Table 5.

Table 5. USG Analysis

Masalah	USG Score			Total Score	Priority
	U	S	G		
Student Misconceptions in Adding and Subtracting Fractions	5	4	4	14	Priority 1
The Issue of Using Two Languages in Mathematics Education in Integration Cambridge Curriculum and National Curriculum	3	4	2	9	Priority 2
The Issue of Mathematics Learning Media	2	2	1	5	Priority 3

The scoring in USG analysis is evaluated and quantified using a Likert scale in Table 6.

Table 6. Likert Scale

Score	Category
5	Priority Important
4	High Priority
3	Medium Priority
2	Low Priority
1	Not A Priority

(Source: Adaptation from Naser et al. (2022))

The position of this research concerning previous research conducted by experts in the field of mathematics education is presented in Table 7.

Table 7. Research Position

Reasercher (Year)	Tittle	Subject	Research Focus	Result
Viviana et al. (2019)	<i>Analisis Miskonsepsi Siswa pada Operasi Penjumlahan dan Pengurangan Pecahan di Kelas V Sekolah Dasar</i>	5 th grade students	Studying and describing misconceptions and their causes.	Misconceptions experienced by students include adding numerators and denominators together.
Sadiyah & Afriansyah (2023)	<i>Miskonsepsi Siswa Ditinjau dari Tingkat Penyelesaian Masalah pada Materi Operasi Pecahan</i>	7 th grade students	Identifying types of misconceptions and understanding causes of misconceptions in solving fraction operations.	Misconceptions of students include generalization, calculation, and notation.
This current recesearch	<i>Case Study of Students' Misconceptions in Adding and Subtracting Fractions and Their Solutions</i>	5 th grade students	Analyze the causes of student misconceptions in addition and subtraction of unlike denominators and their solutions based on fishbone diagrams.	There are nine reasons for misconceptions in addition and subtraction of fractions with unlike denominators, as viewed from the aspects of man, machine, method, material, measurement, and environment.

Based on Table 7, the study conducted by Viviana et al. (2019) aims to explore and describe the various forms of student misconceptions and their causes, with a focus on addition and subtraction of mixed fractions, conducted in the city of Pontianak. The research conducted by Sadiyah & Afriansyah (2023) aims to identify types of misconceptions and understand their causes, with a focus on fraction arithmetic operations in the district of Garut.

This research differs from previous studies, considering subjects, focus, research outcomes, and location. The subject of this study is fifth-grade students at SD Laboratorium UM. This research focuses on students' misconceptions in addition and subtraction of fractions with unlike denominators and their solutions. The distinguishing factor in this study is the application of the APKL and USG methods in selecting the topic to be discussed. Furthermore, this research utilizes a fishbone diagram to analyze priority topics and provide solutions for each problem cause identified in the diagram.

If researchers don't analyze the causes of student misconceptions in addition and subtraction of fractions with unlike denominators through a fishbone diagram and seek solutions, then the subject may experience persistent misconceptions. Therefore, this study aims to analyze the causes of student misconceptions in addition and subtraction with different denominators, along with their solutions based on a fishbone diagram.

METHOD

This research was a qualitative study with a collective case study design that examines issues from several events collectively in a single study (Candrama et al., 2023; Susiswo et al., 2024). The issue investigated in this study is the analysis of students' misconceptions in addition and subtraction of fractions with unlike denominators using a fishbone diagram.

The research location is at SD Laboratorium UM because it has a sufficient number of students who are representative of selecting research subjects. Additionally, the researchers have obtained permission from the school to conduct the research. Furthermore, the school location is easily accessible for the researchers, facilitating the data collection process and interaction with students and teachers.

From 24 students, the researchers chose three fifth-grade students with a learning experience in fraction operations. Students' learning experience is a sufficient condition for analyzing student misconceptions because it serves as a trigger in solving problems (Darmawan, 2019). The research subjects were selected

through the snowball sampling technique. This selection was done in such a way that saturated data was obtained (Darmawan & Yusuf, 2022). The characteristics of the research subjects are presented in Table 8.

Table 8. The Characteristics of Research Subjects

Subject Characteristics	Reasons for Subject Selection
Students who experience misconceptions in addition and subtraction fractions with unlike denominators	The causes of student's misconceptions are important to comprehensively studied and solutions should be sought to prevent ongoing problems.

The research instruments include the researchers, written test, semi-structured interview guide, audio recording equipment, and researchers notes. Researchers present data from one subject because it represents data from other subjects. The data sources for this research are (1) the subject's written answer, (2) the subject's interview, (2) the mathematics teacher (Code: N2), (3) the head of facilities and infrastructure (Code: N1), and (4) literature from articles and mathematics books. The data sources include written math answers from students and semi-structured interview recordings from students, mathematics teachers, and the head of facilities and infrastructure. Through these data sources, the researchers can identify the causes of student misconceptions in addition and subtraction of fractions with unlike denominators and find solutions to prevent these misconceptions. The research procedure is presented in Figure 4.



Figure 4. Research Procedure

This research utilizes 11 steps of research procedures. The first step is collecting current literature related to students' misconceptions in mathematics. The second step are observing and mapping issues in the literature through a preliminary study, resulting in a problem formulation framework. The third step is to formulate a research problem, which is the cause of students' misconceptions based on a fishbone diagram. The fourth step is choosing the issue to be investigated, which is students' misconceptions in adding and subtracting fractions with unlike denominators. The fifth step is creating a research framework in the form of a flowchart using the APKL, USG, and fishbone diagram methods. The sixth step is creating a research instrument consisting of four types, (1) a written test on addition and subtraction of fractions with different denominators, (2) a semi-structured interview guide, (3) research notes, and (4) audio-visual equipment. The seventh step is to validating instruments by a doctor in the field of mathematics education. The eighth step is to refining the research instrument by revised it according to the validation results. The ninth step involves conducting research in two stages, (1) administering a written test on addition and subtraction of fractions with different denominators in the form of essay questions simultaneously to potential subjects, and (2) selecting research subjects based on their test answers and elaborating on those answers through interviews to obtain information related to student misconceptions. Tenth step, analyzing data by categorizing it into one category of misconceptions, then analyze the causes

The qualitative data analysis technique used in this study is interactive (Miles & Huberman, 1994) in Figure 5.

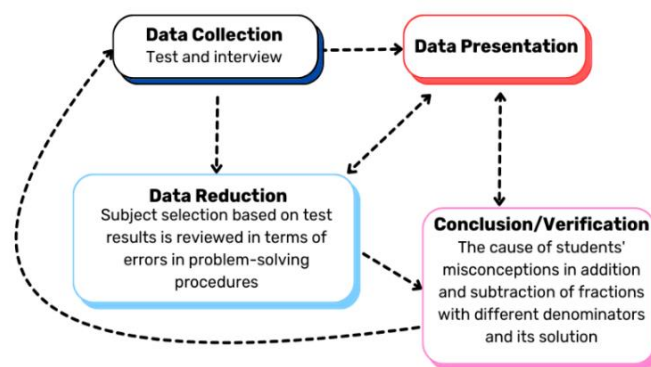


Figure 5. Interactive Data Analysis

Data collection through tests and interviews. Researchers administer written tests to potential subjects. The collected data is analyzed using the applicable theorem in Figure 1. Data reduction is done by selecting subjects who have misconceptions. The researchers further elaborate on test answers through interviews with students, mathematics teachers, and the school's facilities and infrastructure coordinator. Data presentation is done by presenting written answers and interview results, which are then elaborated upon to describe the factors causing misconceptions in subjects and solutions to address them. Lastly, researchers conclude student misconceptions and solutions to overcome them.

RESULTS AND DISCUSSION

Misconceptions occur due to certain factors. The same applies to misconceptions about adding and subtracting fractions, unlike denominators. The factors underlying the occurrence of misconceptions in this concept are presented in the fishbone diagram in Figure 6.

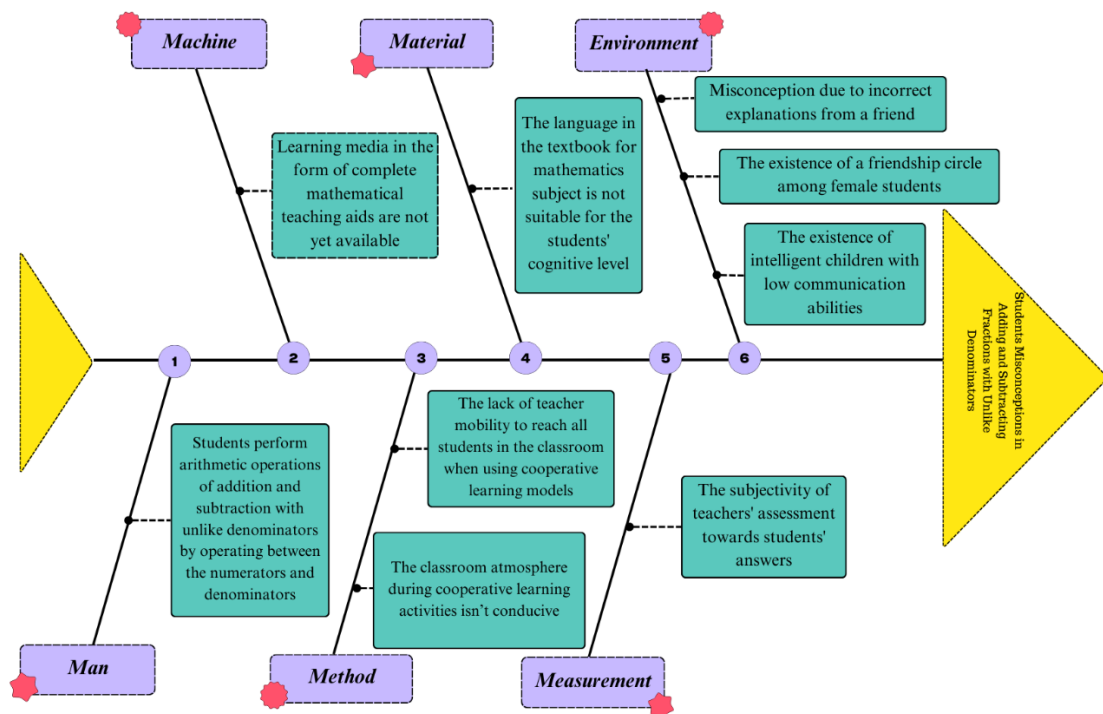


Figure 6. Fishbone Diagram

Man

The cause from man's aspect is that students make mistakes in adding and subtracting fractions, unlike denominators in Figure 7.

$$\frac{3}{2} + \frac{1}{4} - \frac{2}{6} = \frac{2}{0}$$

Figure 7. Student's Mistake

In Figure 7, student is indicated have misconception because they don't understand procedures of these arithmetic operations based on applicable theorems. This indication is further supported by Interview 5.

Interview 5. Confirmation Student Answers

Researcher : "What does $\frac{3}{2} + \frac{1}{4} - \frac{2}{6}$ mean?"

Student : "3 + 1 = 4 and then 4 - 2 = 2"

Researcher : "So, this involves operations between numerators and denominators?"

Student : "Until the subtraction is complete".

Researcher : "So, this also means adding 2 + 4 - 6 for the denominators, right?"

Student : "Yes"

Misconceptions among students in adding and subtracting fractions with unlike denominators arise from a lack of understanding of the applicable theorems, as revealed in the bold-printed statements at Interview 5. Students calculate between numerators and denominators, resulting in the following solutions: $\frac{3}{2} + \frac{1}{4} - \frac{2}{6} = \frac{3+1-2}{2+4-6} = \frac{2}{0}$. The procedure according to theorem is presented as follows: $\frac{3}{2} + \frac{1}{4} - \frac{2}{6} = \frac{(3(4)+1(2))}{2(4)} - \frac{2}{6} = \frac{(12+2)}{8} - \frac{2}{6} = \frac{14}{8} - \frac{2}{6} = \frac{14(6)-2(8)}{8(6)} = \frac{84-16}{48} = \frac{68}{48} = \frac{17}{12}$.

Machine

From machine aspect, the lack of teaching aids in learning activities is due to abstract nature of mathematical concepts, which require the assistance of teaching aids. The classroom condition in Figure 8 shows that teaching aids for addition and subtraction of fractions aren't available.



Figure 8. The Classroom Condition

The indication was comprehensively explored by researchers during interviews with the Deputy Head of Facilities and Infrastructure (N1) and the Elementary School Mathematics Teacher at UM Laboratory (N2) in Table 9.

Table 9. Confirmation of Causes Student Misconceptions from Machine Aspect

Interview N1		Interview N2	
Researcher	: <i>“What are the factors behind the unavailability of teaching aids in the classroom?”</i>	Researcher	: <i>“Have you ever used manipulative media in learning?”</i>
N1	: <i>“The abundance of materials is a challenge, so it’s not always possible to demonstrate or create specific media”</i>	N2	: <i>“I used paper as a medium. I folded the paper in half, then folded it again. Yesterday, I used the same medium, but colored it for basic fraction materials”.</i>

In Table 9, indication of student misconceptions is due to the lack of validated teaching media. This is because mathematics teachers only use teaching aids for complex topics. However, for adding and subtracting fraction with unlike denominators don’t have teaching aids provided. Even though, these topics require visual aids to convey abstract concepts to students.

Method

From method aspect, researchers found two issues in implementation of cooperative learning models with peer tutors: (1) the lack of teacher mobility to reach all students in the classroom, and (2) the less conducive classroom atmosphere during learning activities. The interview results are presented in Table 10.

Table 10. Confirmation of Causes Student Misconceptions from Method Aspects

Problem 1	
Researcher	: <i>“What is the role of peer tutors in learning?”</i>
N2	: <i>“Every group has a leader. When there is a lesson or assignment, it can be discussed together, so the tutor acts as a bridge between the teacher and the students. Because I cannot explain individually, it will be the group members who explain to their peers”.</i>
Researcher	: <i>“Does giving incorrect explanations to fellow students result in misconceptions for the ones being taught?”</i>

N1	: “Yes, because student with low academic abilities usually just follow along”.
Problem 2	
Researcher	: “How is the atmosphere during the implementation of cooperative learning models?”
N2	: “Students are more actively receiving explanations from their peers, but they are definitely noisy when it comes to elementary school children”

The indication of inappropriate implementation of teaching methods by mathematics teachers as factor of misconceptions in students is revealed in the bold statements in Table 9. Because, in the application of cooperative learning models, teachers cannot reach all students in the classroom. Teachers rely on peer tutors as intermediaries for explaining concepts to other students. However, not all peer tutors have good communication skills and may make mistakes in explaining concepts. More than that, the classroom conditions in cooperative learning tend to be non-conducive, which can hinder student’s thinking process and impact their understanding of mathematical concepts.

Material

In material aspect, the use of language in textbooks for mathematics subjects isn’t suitable for student’s cognitive levels is indicated as a factor in student’s misconceptions in adding and subtracting fractions with unlike denominators. This indication in Figure 9.

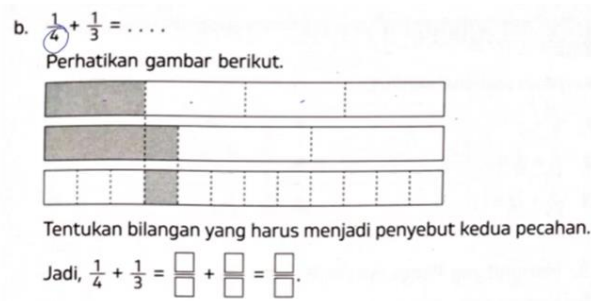


Figure 9. Addition Fractions in Mathematics Book

In Figure 9, the language used to convey the problem-solving instructions isn’t clear, because it only states the command. Determine the number that should be the denominator of the second fraction. However, students should actually be

instructed to shade the rectangular area first. As a result, students are confused in solving the problem and it leads to misconceptions. This indication is supported by Interview 6.

Interview 6. Confirmation of Causes Student Misconceptions from Material Aspects

Researcher : “The language used in the Erlangga Book and the Cambridge Math Book is not suitable for student’s cognitive level, right? Because it’s too advanced?”

N2 : “The Cambridge one is easy, but it’s expensive”

Researcher : “If the book from Erlangga is also not suitable for students’ cognitive level due to its high language?”

N2 : “**Yes, the language is too high, it’s too difficult.** In my opinion, it’s not about HOTS, but just difficult questions, because HOTS questions don’t always have to be difficult”

The indication causes of misconceptions from a material aspect is revealed in the bolded statement in Interview 6. The book used a high-level language, which is not suitable for the students’ cognitive level.

Measurement

From measurement aspect, the subjectivity of teacher assessments of student answers appears to be a contributing factor to student misconceptions. In administering tests, mathematics teachers typically use multiple-choice, essay, and open-ended questions. To understand about assessment system in tests and uncover these indications, interviews were conducted with mathematics teachers in Interview 7.

Interview 7. Confirmation of Causes Student Misconceptions from Measurement Aspects

Researcher : “What assessment system is used? Holistic, analytical, or something else?”

N2 : “Usually we use an established analysis with predetermined values, the assessment rubric is already in place. For example, multiple choice is 1, essay is 2, and open-ended questions are 3”

Researcher : “Are there specific indicators for determining scores?”

N2 : “**For the merdeka curriculum final exam, for instance, for question numbers 1 and 2, how many TP (correct answers) are achieved will be visible**”

The indication is revealed in the bold-print statement in Interview 7 because the assessment indicators in the research rubric aren’t specific, where the indicators depend on the achievement of TP in each question. From that statement, it is implied that there are no specific indicators like in the holistic and analytical

assessment rubric, which leads to subjectivity in evaluation. Therefore, the cause of errors in the measurement aspect is the subjectivity of teacher assessment towards student answers.

Environment

From environmental perspective, researchers examine the causes of misconceptions through interviews presented in Table 11.

Table 11. Confirmation of Causes Student Misconceptions from Environment Aspects

Problem 1	
<i>Researcher</i>	<i>: "If a student gives incorrect explanations to their friend, does it result in misconceptions for the friend being taught?"</i>
<i>N2</i>	<i>: "<u>Yes, usually students with low academic abilities just follow along</u>"</i>
Problem 2	
<i>Researcher</i>	<i>: "Is there segregation for students with learning disabilities?"</i>
<i>N2</i>	<i>: "[If it's isolation, no, but sometimes those girls if they don't like it, they become lazy to approach]"</i>
Problem 3	
<i>Researcher</i>	<i>: "What are the criteria for peer tutors?"</i>
<i>N2</i>	<i>: "From the results of learning, communication abilities, and a child's character. <u>Because a smart child may not necessarily want to explain things to their friends Sometimes, smart children are not well-liked by their peers because their ego is high</u>"</i>

The factors causing student misconceptions from environmental aspect are revealed in the statements in Table 11. There are three problems that contribute to these factors. The first problem is indicated by the underlined statement in Problem 1 of Table 11, which states that students acquire misconceptions from explanations given by their peers. The second problem is highlighted by the [] statement in Problem 2 of Table 11, which shows that students are unable to collaborate with their peers due to friendship issues. The third problem is indicated by the bold and underlined statement in Problem 2 of Table 11, which states that intelligent students may not necessarily have the skills to be peer tutors. This is because intelligent students often have high egos, poor communication abilities, and a fear of losing to their peers, leading them to be unwilling to share their knowledge with others.

Discussion

Based on the factors causing student's misconceptions, there are several solutions that can be applied to address these misconceptions, considering aspects such as man, machine, method, material, measurement, and environment.

Man Aspect Solution

The solution from man aspect through teachers is teachers explain the theorem of adding and subtracting fractions with unlike denominators concretely. This is because fifth-grade students are in the age range of 10-11 years, thus entering the concrete operational stage in cognitive development according to Piaget (Marinda, 2020). At that stage, children have ability to think logically about concrete events. Therefore, teachers can create visual representations related to these concepts by connecting them to everyday problems (Lestari, 2022).

Machine Aspect Solution

The solution from machine aspect is through utilization visual aids or concrete learning media. This media can be accessed by all students in the classroom and uses objects found around them. The visual aid 'fraction addition and subtraction board' proposed by Fatimah (2023). Effectively used in learning and presented in Figure 10.

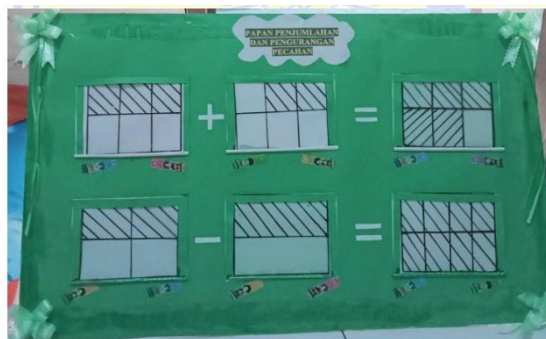


Figure 10. Fraction Addition and Subtraction Board

The equipment used in the teaching aid includes manila paper and black markers to illustrate fraction shapes. Additionally, cardboard and buffalo paper serve as the base for the teaching aid. The fraction addition and subtraction board is equipped with question cards. Teachers can combine the use of this teaching aid

with Student Worksheets (Wahyuni & Darmawan, 2023). The activity aims for students to systematically and gradually learn mathematical concepts according to the mathematical learning theory proposed by Bruner (Supono, 2023). That is, starting from the concrete, semi-concrete, and abstract.

Teaching aids are effective if they can be accessed by all students, therefore it is necessary to form a heterogeneous study group consisting of 5 students. The steps for using the teaching aid are as follows: students choose 1 out of 5 question cards that they receive in their study group. Students draw squares on rectangular paper according to the denominator value. So, for example, if they get a question $\frac{1}{2} + \frac{1}{4} = \dots$, then students can draw 2 squares on the first paper and 4 squares on the second paper. Then, the teacher asks, “*how many squares should you draw on the third paper so that the denominators of both fractions are the same?*”. Through questioning, the teacher guides the students to understand that to equalize the denominators, they can multiply them, which is $2 \times 4 = 8$, according to the fraction addition with unlike denominators theorem by Musser et al. (2011) that, “*let $\frac{a}{b}$ and $\frac{c}{d}$ be any fraction. Then $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$,”. Then guide the students to understand that, value $\frac{1}{2} + \frac{1}{4} = \frac{1(4)+2(1)}{2(4)} = \frac{4+2}{8} = \frac{6}{8}$, where the value $\frac{1}{2}$ equivalent to $\frac{4}{8}$, and $\frac{1}{4}$ equivalent to $\frac{2}{8}$. Therefore, the student can shade in a total of 6 squares on the third paper, which was previously divided into 8 squares.*

Method Aspect Solution

From method aspect, the solution can be applied to minimize student errors is by optimizing the use student-centered learning methods with cooperative learning models. One solution that can be used is to provide a learning contract at the beginning of the learning activity. This is because giving a learning contract can have a positive impact on student’s understanding of the concepts being taught and the procedures they need to follow to complete the learning process (Hikmah et al., 2023). Furthermore, teachers can strive to enhance classroom mobility by moving around during group discussions, assisting students who are struggling in group

discussions, and providing re-explanations at the end of the lesson to prevent misconceptions among students (Irnetti, 2022; Prastyani et al., 2019).

Material Aspect Solution

Adding and subtracting fraction with unlike denominators theorem by Musser et al. (2011) show in Figure 11.

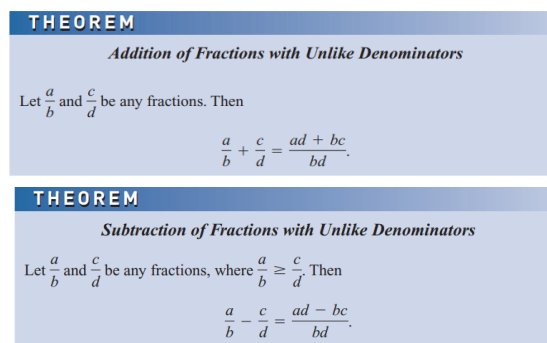


Figure 11. Theorem in Fraction

The theorem in Figure 11 means that the addition of different fractions with different denominators can be done by finding equivalent fractions. For example, $\frac{a}{b}$ and $\frac{c}{d}$ are any fraction, then $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$. Then, for subtraction of fractions, it is also done by finding an equivalent fraction. For example, $\frac{a}{b}$ dan $\frac{c}{d}$ are any fraction, where $\frac{a}{b} \geq \frac{c}{d}$, then $\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$.

According to the Oxford dictionary, it is mentioned as a definition *least common denominator* atau kelipatan persekutuan penyebut terkecil adalah sebagai berikut. “*least common denominator = least common multiple of the *denominators. Used in adding numerical or algebraic fractions efficiently. So, $\frac{5}{12} + \frac{3}{8} = \frac{10}{24} + \frac{9}{24} = \frac{19}{24}$ is simpler than using 96 as the common denominator for the equivalent fractions*” (Clapham et al., 2014).

The theorem and definition indicate that, in adding and subtracting fractions with unlike denominators, equivalent fractions can be found. However, there is a difference between the theorem and the definition. In the theorem, finding equivalent fractions is done by multiplying the denominators. Whereas in the definition, finding equivalent fractions is done by finding the least common multiple (LCM) of those denominators.

Therefore, in teaching the concept, teachers must pay attention to which theorem or definition is being used, how to find equivalent fractions based on the theorem or definition being used, and must be consistent so that students do not become confused in understanding the concept. The concept of addition and subtraction of fractions with different denominators applies the principle that $\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$ is a mistaken concept. That is because, if constructed into geometric shapes, the shape is different. An example of its application in the problem is as follows, “Given the operation of adding fractions $\frac{1}{4} + \frac{1}{3} = \dots$, describe the operation in the form of squares”.

a. $\frac{1}{4} + \frac{1}{3} = \frac{3}{12} + \frac{4}{12} = \frac{7}{12}$

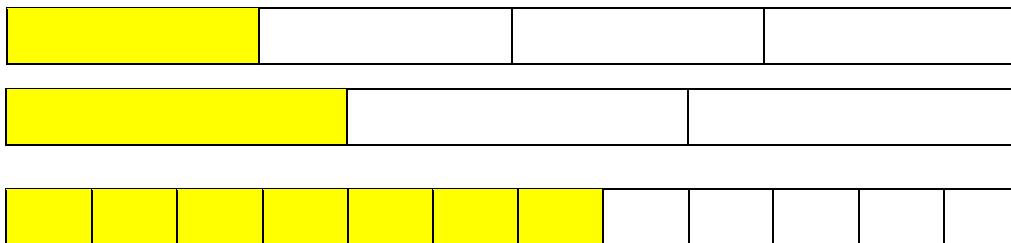


Figure 12. Incorrect Representation in Geometric Form

b. $\frac{1}{4} + \frac{1}{3} = \frac{2}{7}$

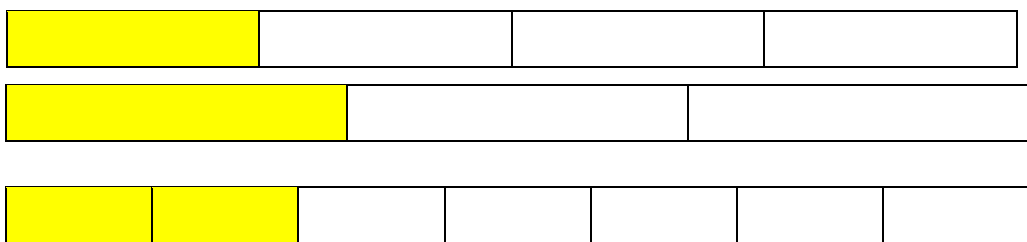


Figure 13. Correct Representation in Geometric Form

From Figure 12 and Figure 13, if the student answers by performing the operation $\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$, then representation in geometric form of $\frac{1}{4} + \frac{1}{3} = \frac{2}{7}$ This does not match the hatched area, where the hatched area should be larger than the hatched area of $\frac{1}{4}$ or $\frac{1}{3}$. Therefore, the correct method is to first find the equivalent fractions by using the least common multiple (LCM) or multiplying the denominators.

According to the Student Mathematics Book for Grade V Elementary School published by the Ministry of Education and Culture, examples of material and the appropriate teaching methods for instilling the concept of addition and subtraction of fractions with unlike denominators have been explained. The teaching material examples are presented in Figure 14.

Eksplorasi 3.2D
Masalah Lukas
 Lukas mempunyai sebagian kertas. Kemudian, Komang memberi Lukas sebagian kertas lagi. Apabila kedua bagian kertas tersebut digabungkan, dapatkah kamu menyebutkan pecahan dari gabungan kertas tersebut?

Cara Nisa
 Amatilah gambar berikut ini.
 1. Kertas Lukas semula
 $\frac{1}{3}$
 Hal ini senilai dengan $\frac{2}{6}$

2. Kertas yang diberikan Komang ke Lukas $\frac{4}{6}$
 Sehingga ketika digabungkan, $\frac{2}{6} + \frac{4}{6}$
 Jadi, gabungan kertas tersebut adalah $\frac{2}{6} + \frac{4}{6} = \frac{6}{6}$

Cara Yohana
Penjumlahan Pecahan dengan Penyebut Berbeda
 Contoh $\frac{1}{3} + \frac{4}{6} = \dots$
 Penyelesaian $\frac{1}{3} + \frac{4}{6} = \dots$
 Mencari KPK dari 3 dan 6.
 Kelipatan 3 adalah 3, 6, 9, 12, 15, 18, 21, 24, ...
 Kelipatan 6 adalah 6, 12, 18, 24, ...
 KPK dari 3 dan 6 adalah ...
 Jadi,
 $\frac{1}{3} + \frac{4}{6} = \frac{(6:3) \times 1}{6} + \frac{4}{6}$
 $= \frac{2}{6} + \frac{4}{6} = \frac{2+4}{6} = \frac{6}{6}$

Figure 14. Example of Material Taught

Material in figure 14, can be adopted by teachers during learning activities to help students interpret the concept more easily. Students will understand that to solve problems related to this concept, they need to first find equivalent fractions by either finding the least common multiple (LCM) of the denominators or multiplying the denominators, and then follow the applicable rules.

Measurement Aspect Solution

From measurement aspect, the subjectivity of teacher assessments of student answers needs to be overcome. The solution to this issue is to use an assessment rubric with specific indicators. For students, understanding these rubrics is crucial so that they can maximize their performance when completing tasks, exams, and other assessments according to the indicators specified in the rubric. There are two types of assessment rubrics, holistic and analytic (Mahendra, 2019). The holistic rubric contains levels of work that describe the quality and quantity of student work, while the analytic rubric includes detailed assessments for each criterion (Suwarno & Aeni, 2021). Through the use of holistic or analytical sections

for essay questions, it is hoped that the subjectivity of previous teacher assessments can be reduced.

Environment Aspect Solution

From environmental aspect, first teachers can provide an introduction at the beginning of the learning activity, give feedback on group discussion outcomes, and offer personalized approaches to students who need clarification regarding their misconceptions (Mariani, 2021; Raharjo et al., 2021). Secondly, the teacher advises students on how to behave well in making friends (Khaira et al., 2023). Thirdly, the teacher advises students to help each other by sharing knowledge with their peers (Lubis et al., 2021). As a result, student's fear of competition among high-achieving students can disappear, and they will be willing to teach their peers, leading to an equal distribution of learning outcomes in the good category.

CONCLUSION

The research findings indicate that student's misconceptions in addition and subtraction of fractions with unlike denominators are caused by several factors based on the fishbone diagram. These factors include human error in calculation operations involving numerator and denominator without equalizing the fractions, the absence of visual aids for different denominators, limited teacher mobility in teaching, non-conducive classroom atmosphere, irrelevant language in textbooks to students' cognitive level, subjective assessment by teachers, peer tutors providing incorrect explanations, specific friendship groups, and high-ability students with low communication skills.

This research has limitations; namely, the study is limited to elementary school scope, the solutions presented are the result of literature analysis and have yet to be concretely tested by researchers, so their effectiveness cannot be measured. In addition, the concept studied is limited to addition and subtraction fractions, unlike denominators. Recommendations for further research should be expanded to higher levels and other mathematical concepts. Furthermore, the researchers recommend developing tools or learning strategies that minimize students' misconceptions about mathematical concepts for future research.

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