

## Designing a Learning Trajectory of Area and Perimeter of Flat Shapes with Realistic Mathematics Education Approach for Fourth Grade Elementary School

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**Abstrak.** This study was motivated by the lack of students' understanding of the concept of learning mathematics in elementary schools in measuring the area and perimeter of flat shapes. Therefore, the researchers designed and developed a learning trajectory of area and perimeter of flat shapes, square and rectangle, with Realistic Mathematics Education approach. The study used design research as methodological approach consisting of three phases, the preparatory phase, the experimental phase, and the retrospective analysis phase. The learning trajectory was tested with 14 students, 4 students for one to one and 10 students for experimental process. The data were gathered from interviews with several students, observations from videos during the learning process, and analysis of students' worksheets. The findings of the study showed that the learning trajectory of area and perimeter of flat shapes with Realistic Mathematics Education approach triggered students to develop their mathematical reasoning on the concept of area and perimeter of flat shape, square and rectangle.

**Kata kunci:** Area and Perimeter; Design Research; Flat Shape; Realistic Mathematics Education

### INTRODUCTION

Flat shapes are an essential topic in primary school mathematics. Flat shapes are examined in the 2013 curriculum beginning in second grade (Lisnani, 2017). Students start introducing flat shapes including squares, rectangles, parallelograms, trapezoids, rhombuses, kites, and triangles (Lisnani, 2017). Learning flat shapes is typically taught by explicitly giving the formula for calculating the area and perimeter of a flat shape, thus students do not comprehend the notion of how the formula emerges (Zacharos, 2006). This difficulty arises throughout the course of students' grasp of ideas linked to measuring the area and perimeter of flat shape.

Comprehensive ideas are difficult for primary school kids to understand (Puspasari et al., 2015). Learning implementation just educates students to answer issues with broad formulae without teaching them how the formulas are generated

(Winanda et al., 2020). School materials also place a greater focus on processes, including formulae, for calculating the area and perimeter of flat objects like squares and rectangles. According to the findings of a research done by Wijaya et al. (2015), the proportion of relevant learning opportunities in junior high school mathematics books is relatively low. This undoubtedly contributes to students' lack of comprehension of the ideas and strategies necessary to solve mathematical issues, particularly the area and perimeter of flat objects (Pratiwi, 2018). Aside from that, Zacharos (2006) claims that students are still puzzled about grasping the ideas of area and perimeter when it comes to addressing issues provided by students who prefer to remember formulae, which is inversely proportionate to students who understand the concepts.

There are various approaches that can be applied to support students' abilities and skills in learning flat shapes. One of these approaches is Realistic Mathematics Education (RME) (Freudenthal, 1991; Treffers, 1991). RME is a mathematics learning theory which emphasizes that learning must be delivered to students in concrete or real-life situation. This theory is was first developed in the Netherlands by Freudenthal (Freudenthal, 1983). This theory has also been developing in Indonesia for more than 20 years, and it affects the educational policy in this country until nowadays (Sembiring et al., 2008).

There are 6 RME learning principles, namely activity, reality, hierarchy, interconnection, interaction and guidance principles (van den Heuvel-Panhuizen & Drijvers, 2014). The activity principle means that learning mathematics is an activity carried out by students to understand both mathematics as a science and mathematics to be applied in real-life or other problems. The reality principle means that mathematics should be learned from something real for students, especially in elementary school. Hierarchy principle means that to arrive at an abstract mathematics, students need a process that starts from concrete and semi-concrete things. The interconnection principle means that learning one mathematics topic is related to other topics, such as learning measurement is related to students' understanding of numbers and algebra. The interaction principle means that when studying mathematics, students are required to interact with each other, among

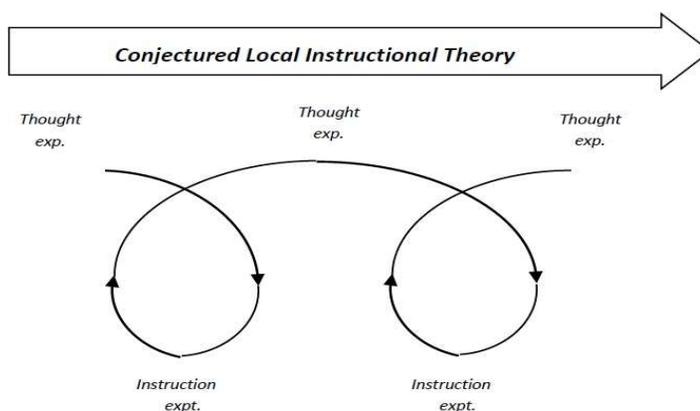
students and with the teacher. Lastly, the guidance principle means that mathematics learning really requires the teacher's role in guiding students to discover the mathematical concepts being studied.

There are several previous studies that have applied RME in mathematics learning in elementary schools. Putra et al. (2011) designed learning about number facts up to 10 using the context of parrots, namely the Parrot game. This learning was able to stimulate students to learn numbers in a fun way and develop their number sense. Rahmayani et al. (2021) also designed learning the volume of flat-sided spatial shapes using RME. The results of this research show that students were able to build their knowledge about how the formula for calculating the volume of geometric shapes emerges from the learning activities carried out. Yuberta et al. (2011) designed learning to measure the area of flat shapes using RME and showed that students had the opportunity to build their knowledge through RME learning. However, the real context designed must be in line with the characteristics of students. Therefore, researchers are interested in designing a learning trajectory of flat shapes using the RME approach in grade 4 elementary school. The aim of this research is to find out how students build their knowledge about the area and perimeter of squares and rectangles through the RME learning approach.

## **METHODS**

The research method used in this study is design research. Design research is systematic research that designs, disseminates & evaluates educational hegemony in the form of programs, strategies, learning materials, products and systems as solutions to complex problems in educational practice (K. Gravemeijer & Cobb, 2006). Design research was chosen as the appropriate approach to achieve the objectives of this research because it is in accordance with our objective, namely developing a learning path for the area and perimeter of a flat shapes.

In this research, we focused on developing learning trajectory as an initial phase of preparation for local instructional theory, considering that this work was limited to only two cycles (Figure 1). The design research approach used in this research is based on the views of Gravemeijer and Cobb (2006) regarding design research for mathematics learning. In this case, design research focuses on the learning process or learning exploration carried out by students from the learning environment being developed.



**Figure 1. Design Research Process**

Design research consists of three phases namely: a). Preparing for the experiment, b). Implementation of pilot and teaching experiments, and c). Retrospective analysis. In the first phase, we carried out a literature review by collecting information on learning materials, strategies, and learning objectives in accordance with the RME approach. We develop learning trajectory that are dynamic, meaning they can be modified and adapted to students' situations during the experimental process.

In the second phase, we conducted pilot and teaching experiment. This phase used field notes as a tool to identify interesting events in the learning process. In the pilot experiment, the first researcher played the role of teacher. The goal of the pilot experiment is to coordinate the sequence of activities and what is developed and improved to better design in the next cycle. Teaching experiments were carried

out with small groups in which all students participated. The homeroom teacher acts as the teacher and the first researcher acts as an observer of learning activities.

The third phase is retrospective analysis. At this stage, the data obtained from carrying out experiments is analysed using a hypothetical learning trajectory which is claimed to be a reference for determining the focus of the analysis. We watched videos of learning activities and used field notes as information about what was done or not done from the assumed learning, while showing the assumed learning flow during the learning process, and the students' actual learning process. In addition, based on the results of the analysis, conclusions are drawn that describe the student's learning flow.

The subjects of this study were 14 fourth grade students from a public elementary school in Kampar District, Riau, Indonesia. This study involved 4 students in the pilot experiment and 10 students in the teaching experiment. In this study, we designed a learning process known as hypothetical learning trajectory (HLT). HLT is needed because researchers need to predict what will happen later in class. HLT consists of learning objectives, mathematical activities that support student learning, and hypotheses about student learning (Simon & Tzur, 2004). For example, one of the learning objectives is that students can recognize squares and rectangles based on their properties. The activity is to group flat shapes based on shape and size. The hypotheses are some students can group the objects based on colour, size, and shape.

The instruments of this study are field notes, student worksheets and interview guides (Sugiyono, 2019). The tools used were previously discussed with the research team. Field notes were made when they found interesting moments during the learning process. The results of the worksheet were collected and analysed, and student interviews were conducted in the form of video recordings. Data is recorded and analysed to see if they match previously designed assumptions.

Data collection techniques in this research are observational, documentation (2 cameras, static and dynamic), field notes as additional data, and interviews. Observations are carried out by observing the implementation of the learning process, making observation notes to support data collection through video

recording. The data collection technique in this research is observation which consists of individual and small group experimental observations, observation of the learning process, student-student interactions, and student-teacher interactions recorded via videotape. Crucial moments in the video were transcribed & analysed based on RME principles.

## **RESULTS AND DISCUSSION**

### **Pilot Study**

The pilot study was carried out with 4 students divided into 2 groups. The HTT from this pilot study consists of 5 activities, namely, grouping flat shapes based on properties, finding the area of a square, finding the area of a rectangle, finding the perimeter of a square, and finding the perimeter of a rectangle. The context given in lesson 1 is decorating the class with various flat shapes made from origami paper or coloured paper for Independence Day. Students are then asked to group the shapes before arranging them.

Learning in this pilot study was carried out well, but researchers found several obstacles faced by students in learning. Although students can easily group flat shapes because the flat shapes have the same size and colour, students are hesitant when asked about their reasons for grouping these objects.

Researcher : Have the Lychee Group found the differences and similarities between squares and rectangles?  
Lychee : (Silence)  
Researcher : The Lychee Group knows which sides are in a square and a rectangle?  
RP : This is the side (towards the side of the square)  
Researcher : How many sides does a square have?  
RP : A square has 4 sides  
Researcher : How many sides are there for a rectangle?  
Lychee : (Silence)  
Researcher : where are the sides of the rectangle  
GL : this is a rectangle (towards the side of the rectangular paper)  
Researcher : Well, how many sides are there?  
GL : There are 4 sides ma'am  
Researcher : Apart from the side, what else is there??  
RP : ehmmmm (thinking)  
Researcher : there are corners, how many corners are there in a square and a rectangle??  
GL : ehmm, a square has 4 corners, and a rectangle also has 4 corners.

The conversation above proves that students are actually able to group flat shapes correctly based on the properties, but when answering questions students hesitate to answer them. Besides, we realised that the same colours of the shapes did not challenge them to group the flat shapes. Therefore, we modified the activity by adding more colourful shapes and sizes.

### **Teaching Experiment**

The teaching experiment was carried out by the homeroom teacher as a model teacher, 10 students, and the first researcher as an observer. This cycle is an improvement from the previous cycle, namely a pilot study.

*Activity 1: determine the properties and differences between squares and rectangles.*

Students are divided into five groups, each group consisting of two students. In this lesson, the teacher first explained the properties of square and rectangular shapes, because the teacher did not seem to understand the lesson designed by the first researcher. Then, the teacher presents the task to the students in which they must group some flat shapes before arranging them for decorating the class for Independence Day.

From the results of observations, it was found that students could differentiate between square and rectangular shapes, and they were able to group them according to their shape and properties. This was discovered after the teacher asked the students.

Teacher : Why do you group this one (square) and this one (rectangle) differently?  
SA1 : Yes, ma'am. Because the shapes are different, the square has 4 sides of the same length and the rectangle have 2 sides of the same length.

In cycle 2, activity 1, most students were able to determine the properties of square and rectangular shapes according to the properties they already knew and were able to draw square and rectangular shapes. Each can carry out the worksheet correctly according to the steps.

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*Activities 2 and 3: determine the area of a square and rectangle.*

The activity carried out in this lesson is putting together a puzzle and calculating how many squares are needed so that it fills the available frame and styrofoam and then attaching it. The teacher gives several puzzles and asks students in groups to estimate how many unit squares are needed to close the puzzle. As shown in Figure 2, two students are working together to complete the square that covers the puzzle.



**Figure 2. A group Works to Find the Number of Unit Squares to Cover the Puzzle**

Teacher : Have you found the comparison?

SC2 : Yes, ma'am, we found that the large frame requires 36 puzzle pieces, ma'am, so the comparison between the small frame and the large frame has a difference of 20 pieces, ma'am.

Teacher : Wow great, how did you find it?

SC1 : We put together the puzzle pieces of paper using layers of stickers on the top and sides, ma'am, then we multiplied the side and top sides, ma'am, and we got the results, and we compared them with the small frame.

From lesson 2, students have been able to find and understand the area of a square. Apart from that, they were also able to compare the area between small frames and large frames. Similarly, in lesson 3 students were also able to find the approximate area of a rectangle correctly. It can be concluded from this activity that the entire group already understands how to find the area of a square and rectangle, although there are some students who have not memorized multiplication, but with the help of a unit square and trying to predict the number of units squares they are able to find the area of a square and rectangle.

*Activities 3 and 4: determine the perimeter of a square and a rectangle.*

In activities 3 and 4, students are asked to determine the perimeter of a square and a rectangle by choosing measuring tools that have been provided, namely ropes, rulers and sticks. After they chose the measuring tools, they continued to measure the edge of the frame (square), and the edge of the Styrofoam (rectangle). Figure 3 shows a group of students measuring a side of the Styrofoam.



**Figure 3. A group Measures a Side of the Styrofoam**

Teacher : how about group A, how long of a ribbon do you need?

SA1 : we need 2 pieces of 40 cm ribbon for the wide side and 2 pieces of 59 cm ribbon for the long side, ma'am.

Teacher : why are the sizes different?

SA2 : yes ma'am, because the length of the sides are different ma'am, the side side is 40 cm while the bottom side is 59 cm ma'am.

Teacher : So how many lengths of ribbon do you need?

SA1 : because 2 sides are 40 cm wide it becomes 80 cm, while 2 sides are 59 cm long so it is 118 cm then add  $80+118= 198$  cm ma'am.

From the discussion students already understand how to find the perimeter of a rectangle. Students understand that a rectangle has 2 pairs of parallel sides of the same length so that determining the perimeter of the rectangle can be done by multiplying each side by 2. This shows that learning carried out based on RME can support students to construct their knowledge regarding measuring the perimeter of square and rectangle.

## **Discussion**

Designing a learning trajectory using RME approach can help fourth grade students understand properties, area, and perimeter of square and rectangle. This is shown when students are presented with real life situations that are close to their lives. At the first meeting, this activity provided a context for grouping flat shapes based on their properties. As students do so, students will learn the properties and differences between squares and rectangles. Students begin to move from the visualization stage to the analysis stage in accordance with Van Hiele's theory (Khalil et al., 2018). At this level students begin to see geometry based on its properties. This is in line with the RME principle, namely the level of student understanding, starting with an informal level when solving problems and through various levels and schemes gaining understanding of related concepts and strategies (Gravemeijer, 1994; van den Heuvel-Panhuizen & Drijvers, 2014).

Students in small groups collaborate to find the area of a square and a rectangle. Through interaction, students can build their understanding of the area of a flat shape, namely a rectangle, which is the number of units that cover the surface of the flat shape. These results improve the findings from research conducted by Yuberta et al. (2011) that most students focus on applying formulas to find the area of certain flat shapes without knowing the meaning of area and students do not understand how the formula can be used.

HLT's relationship between measuring area and the perimeter of square and rectangular shapes helps students construct their knowledge. This is in line with the interconnection principle where one learning is related to other learning (Lestari et al., 2019; Sembiring et al., 2008; van den Heuvel-Panhuizen & Drijvers, 2014). Through this, students can find the perimeter of square and rectangular shapes. They were able to find that the perimeter of a rectangle is 2 times the length and 2 times the width. Thus, the learning trajectory runs smoothly from students knowing the properties of squares and rectangles to understanding the area and perimeter of these flat shapes. Learning with RME can build students' better reasoning and conceptual thinking (Iranti et al., 2023; Putra et al., 2011).

## CONCLUSION

The learning trajectory designed regarding measuring the area and perimeter of flat shapes, especially squares and rectangles, based on RME has been able to build students' mathematical knowledge construction skills. Students are bridged from concrete to semi-concrete to abstract mathematics. Students can find the area and perimeter of squares and rectangles. Their understanding is built through problems presented from a real-world context and then guided to discover the properties and differences between squares and rectangles, areas of squares and rectangles, and perimeters of squares and rectangles. However, there are still several obstacles in carrying out teaching experiments, teachers are still confused and awkward, and their understanding of RME is still limited, so that RME-based learning cannot be fully implemented. Therefore, there is a need for RME-based learning training so that teachers can apply and further develop this theory-based learning in the future.

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