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Comparison of Stress-Strain Relationship for Confined Concrete Using Two-Dimensional Fiber-Based Cross-Sectional Analysis

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Abstract: Stress-strain relationship is the main parameter to identify the strength, ductility and behavior of the structure. Various constitutive models were created in order to simplify the analytical approach of concrete behavior. In this paper, the behavior of reinforced concrete column is modeled using Attard and Setunge's (1996) and Mander's (1988) stress-strain constitutive model. The appropriate model for reinforced concrete column was determined based on the existing experimental data. Two-dimensional simulation of reinforced concrete column using fiber-based cross-sectional analysis in MATLAB is sighted. And the performance of the reinforced concrete column from the experimental data is compared with the analysis result from the simulation. There are two comparation methods used in this research. The first method is to compare the linear regression with the reference line. The smallest degree between the linear regression and the referrence line is expected. The second method is to compare the Root Mean Square Defiation (RMSD) value. The smallest RMSD value is expected to get the most suitable constitutive model compared to the experimental data. From the computational process, it was found that Mander's Constitutive model is preferred to be used in further analysis problem concerning reinforced concrete column.

Keywords: Column; constitutive model; reinforced concrete; stress-strain relationship; RMSD.

1. Introduction

It is clearly important to have accurate information concerning the complete stress-strain curve of confined concrete in order to perform a reliable moment-curvature analysis to assess the available ductility of columns with various transverse reinforce arrangements [1].

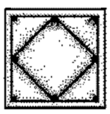
Mander et al [1] model was calibrated using a realistically sized test columns with longitudinal reinforcement and restrained concrete. As a result, the stress-strain relationship based on empirical data is derived. These models depend on the effective area of the column core. Assuming that the column cover will spall off during the test. The scale effect of the tested columns and the presence of the longitudinal reinforcement usually provides the largest estimation of the increase in strength as compared to other research models.

Attard-Setunge's [2] model was determined from a standard triaxial test. The confinement model based on the triaxial test does not observe the assumption of cover spalling. However, the triaxial test replaces confinement of cylindrical specimen with fluid pressure compared to either spiral or column confinement. Fluid pressure becomes active confinement which has a more constant value than passive confinement. Whereas passive confinement depend on the lateral dilatation of the axial load and the characteristics of the confining steel. Richard et al [3] compared the responses between active and passive confinement and found similar behavior. To further investigate the effect of steel fiber on reinforced concrete column, numerical simulation using fiber-based nonlinear sectional analysis in MATLAB were used.

2. Collecting Data

The initial step of the research is by collecting data, which consist of reinforced concrete specifications and axial stress-strain curves from loading test. Table 1 shows spesification of reinforced concrete column collected from the available experimental test in the literature [4-6]. The data consist of axial stress-strain relationship as the result of axial loading test in various rectangular reinforced concrete column. By comparing the experimentally found curves with those estimated by Attard-Setunge's [2] model and Mander's et al [1] model, the most suitable model can be obtained to simplify further research about reinforce concrete column. Therefore, a numerical simulation were performed based on the model and spesification of each column. The simulation using fiber-based nonlinear sectional analysis in MATLAB were used to obtain axial stress-strain relationship. Furthermore, there are three stress-strain relationship can be compared. From the experimental data, also from numerical simulation using Attard-Setunge's [2] model and Mander's et al [1] model.

Table 1. Summary of experimental data from previous researchers.

Researchers	Reinforcement arrangement	Specimen ID	f'_c	Longitudinal Reinforcement			Transverse Reinforcement		
				n	Dia. (mm)	f'_y (MPa)	Dia. (mm)	Spacing (mm)	f'_y (MPa)
Mitchell and Paultre [4]	 235 x 235	5D	99,9	12	20	480	10	65	309
		6D	113,6						
		7D	67,9						
		8D	55,6						
Xiang Zeng [5]	 450 x 450	C-1-A6	25,3	8	24	394	10	72	309
		C-1-A7							
	 450 x 450	C-1-B2		12	20	434			
		C-1-B3							
		C-1-B15	24,8				12	64	296
Sharma et al [6]	 150 x 150	2-C-SC	61,85	4	12	395	8	50	520
		2-C-SA	62,2						412
		2-C-SH	81,8						520
	 150 x 150	2-C-SD	63,35	8					412

3. Processing Data

The next step is by processing data into MATLAB program. In this method, a two-dimensional nonlinear cross sectional analysis with fiber-based model is performed. The analysis was simulated using the computer program code in MATLAB [7]. The reinforced concrete column is loaded under axial concentric compression. It can be run by entering all of the reinforced concrete column spesifications from Table 1 into the program. This program requires a constitutive model to represent the behavior of column sections. The output of this program includes moment-curvature relationship, load-deflection relationship, column interaction diagram and stress-strain relationship that can be extracted from the analysis results. However, in this paper, only the axial stress-strain relationship is obtained from the computational process. It can be used to compare with existing experimental data from various researchers. Based on the previous explanation, there are two types of non-dimensional mathematical models for stress-strain curve of concrete to be compared.

3.1. Constitutive Model by Mander et al [1]

The first one proposed by Mander et al [1] which has general solution for the confined compressive strength as follow:

$$f = \frac{f_o \cdot x \cdot r}{r - 1 + x^r} \tag{1}$$

Where f_o is compressive peak stress of confined concrete from the equation below. And the constants x is given by:

$$x = \frac{\varepsilon}{\varepsilon_o} \tag{2}$$

$$f_o = f'_c \left(-1.254 + 2.254\sqrt{1 + \frac{7.94 \cdot f_r}{f'_c}} - 2\frac{f_r}{f'_c} \right) \tag{3}$$

The f'_c value is obtained from the compressive peak stress of unconfined concrete. And lateral confining pressure (f_r) depends on the configuration of transverse reinforcement. Where ε_o is longitudinal compressive concrete strain from the equation below

$$\varepsilon_o = \varepsilon_c \left[1 + 5\left(\frac{f_o}{f'_c} - 1 \right) \right] \tag{4}$$

And the constants r is given by:

$$r = \frac{E_c}{E_c - E_{sec}} \tag{5}$$

E_c is the tangent modulus of elasticity of the concrete. And E_{sec} is the ratio between f_o and ε_o.

3.2. Constitutive Model by Attard-Setunge [2]

The second one proposed by Attard-Setunge [2] which has general solution for the confined compressive strength as follow:

$$Y = \frac{AX + BX^2}{1 + CX + DX^2} \tag{6}$$

$$Y = \frac{f}{f_o} \qquad X = \frac{\varepsilon}{\varepsilon_o} \tag{7}$$

For the ascending curve under confinement in compression, the constants are given by:

$$A = \frac{E_{ti} \cdot \varepsilon_o}{f_o} \tag{8}$$

$$B = \frac{(A-1)^2}{\alpha\left(1 - \frac{0,45 f'_c}{f_o}\right)} + \frac{A^2(1-\alpha)}{\alpha^2 \frac{0,45 f'_c}{f_o}\left(1 - \frac{0,45 f'_c}{f_o}\right)} - 1 \tag{9}$$

$$C = A - 2 \qquad D = B + 1 \tag{10}$$

E_{ti} is the initial tangent modulus at zero stress. And E_c is the tangent modulus of elasticity of the concrete. Where α is the ratio between E_{ti} and E_c.

For the descending curve under confinement in compression, the constants are given by

$$A = \left[\frac{\varepsilon_{2i} - \varepsilon_i}{\varepsilon_o}\right]\left[\frac{\varepsilon_{2i} E_i}{(f_o - f_i)} - \frac{4\varepsilon_i E_{2i}}{(f_o - f_{2i})}\right] \tag{11}$$

$$B = (\varepsilon_i - \varepsilon_{2i})\left[\frac{E_i}{(f_o - f_i)} - \frac{4E_{2i}}{(f_o - f_{2i})}\right] \tag{12}$$

$$C = A - 2 \qquad D = B + 1 \tag{13}$$

$$E_i = \frac{f_i}{\varepsilon_i} \qquad E_{2i} = \frac{f_{2i}}{\varepsilon_{2i}} \tag{14}$$

Where E_i is the tangent modulus at the first inflection point. And E_{2i} is the tangent modulus at the second inflection point. The stress and strain at the first inflection point defined by f_i and ε_i. The stress and strain at the second inflection point defined by f_{2i} and ε_{21}. The value of compressive peak stress (f_o) of confined concrete can be calculated from the equation below:

$$\frac{f_o}{f'_c} = \left(\frac{f_r}{f_t} + 1\right)^k \tag{15}$$

The f'_c value is obtained from the compressive peak stress of unconfined concrete. The lateral confining pressure (f_r) depends on the configuration of transverse reinforcement.With f_t is the tensile strength of concrete. And the constant k is:

$$k = 1.25\left[1 + 0.062 \cdot \frac{f_r}{f'_c}\right](f'_c)^{-0.21} MPa \tag{16}$$

3.3. Run MATLAB Program

Furthermore, run the MATLAB program, based on the spesification of each specimen ID and mathematical models, to obtain the stress-strain curve of the reinforced concrete column. For example, experimental data with specimen ID 2-C-SC whose concrete compressive strength of the reinforced concrete column was 61,85 MPa. The reinforced concrete column is loaded under axial concentric compression. The cross-sectional shape of the column is a square column with 150 mm width. The concrete cover is set to 10 mm based on the existing spesifications. The number of longitudinal reinforcement used is set to 4 with 12 mm of diameter. The pitch spacing of the transverse rebar is set to 50 mm. The diameter of the transverse rebar is 8 mm with the total leg number in both x and y direction is 2. The longitudinal reinforcing bar have a yield strength (f_y) of 395 MPa. And the lateral reinforcing bars has a yield strength (f_y) of 520 MPa. Fig .2 shows the discretized cross section of the reinforced concrete column meshed with the constant strain triangle (CST) element.

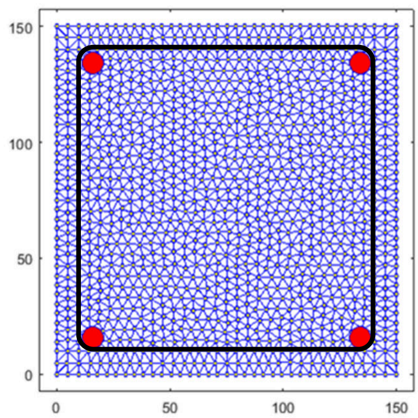


Fig. 1. Discretized cross section of the reinforced concrete column used in the simulation.

4. Compare and Choose the Best Model for Column

There is an existing stress-strain relationship for each data in Table. 1. And right after the computation process is complete, axial stress-strain relationship can be obtained and compared with the experimental data. Stress-strain curves in Fig. 2 to Fig. 8 represents a comparison between three axial stress-strain relationship for selected specimen ID. For example the compressive peak stress of confined concrete (f_o) from experimental data, with specimen ID 2-C-SC, is 73,5 MPa and the axial strain is 0,004. From the analysis as shown in Fig. 6, the compressive peak stress from Attard-Setunge's [2] model is 75,7 MPa which is higher than experimental result. And the compressive peak stress from Mander's et al [1] model is 72,9 MPa which is slightly has the closest value to the experimental result. In general, comparison of stress-strain curve between experimental data from all speciments ID and two models is shown in Fig. 2 to Fig. 8 as follows.

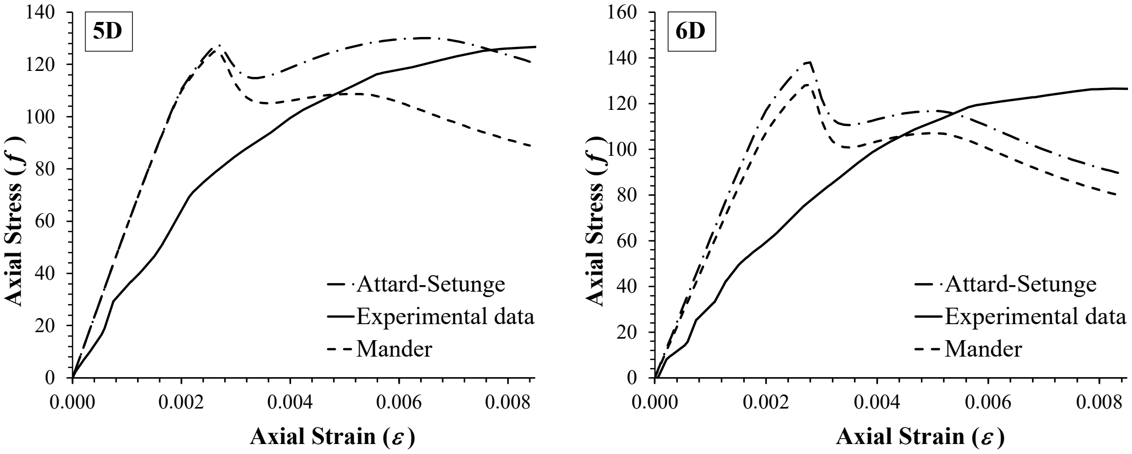


Fig. 2. Axial stress-strain curve for specimen ID 5D and 6D

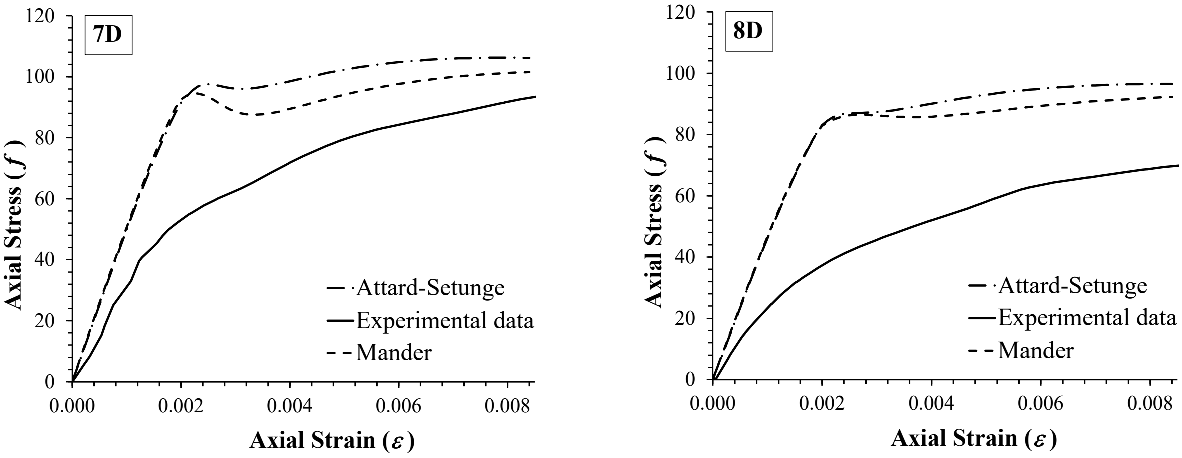


Fig. 3. Axial stress-strain curve for specimen ID 7D and 8D

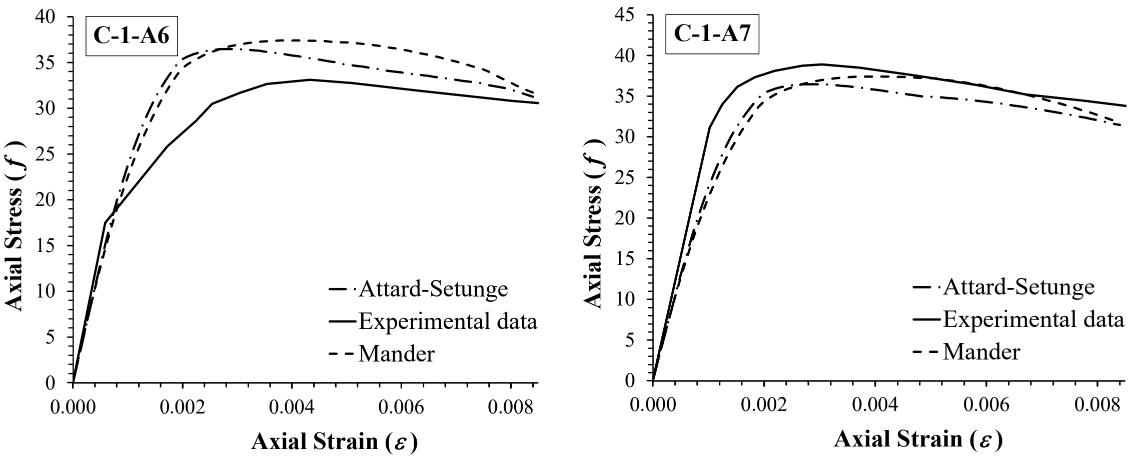


Fig. 4. Axial stress-strain curve for specimen ID C-1-A6 and C-1-A7

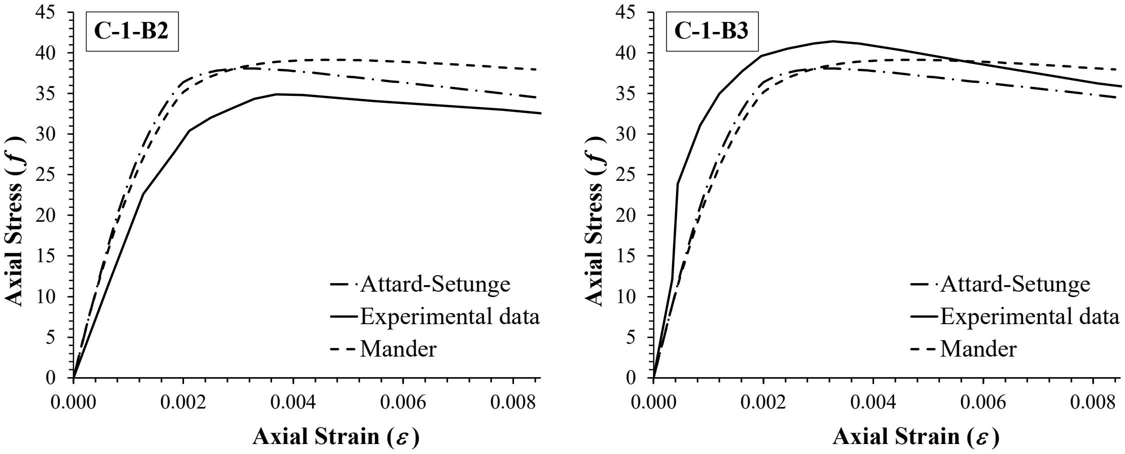


Fig. 5. Axial stress-strain curve for specimen ID C-1-B2 and C-1-B3

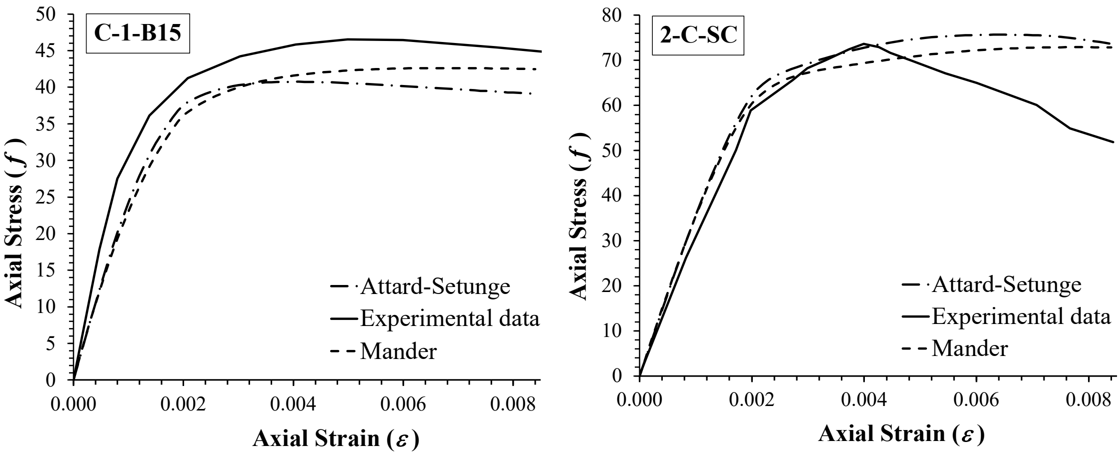


Fig. 6. Axial stress-strain curve for specimen ID C-1-B15 and C-2-SC

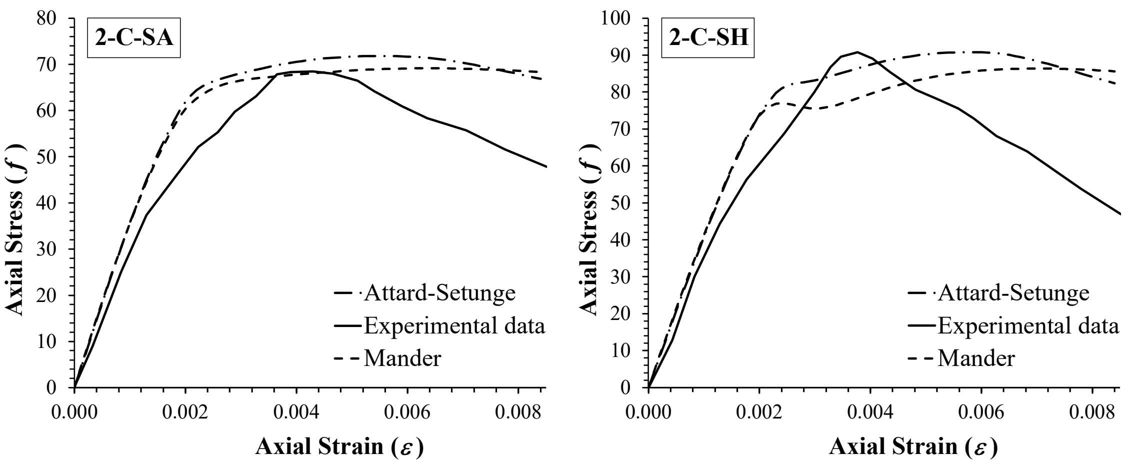


Fig. 7. Axial stress-strain curve for specimen ID 2-C-SA and 2-C-SH

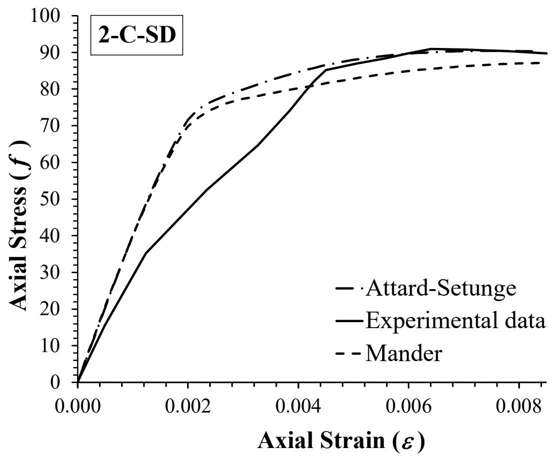


Fig. 8. Axial stress-strain curve for specimen ID 2-C-SD

Comparison between the value of axial peak stress (f_o) experimental data of all speciments ID and both models are summarized in Table. 2.

Table 2. Comparison of the Peak Stress from experimental data and both proposed model

Experimental Data		Model	
Specimen ID	**f_o (MPa)**	**f_o (MPa) Attard-Setunga**	**f_o (MPa) Mander**
5D	127.7	130.0	124.9
6D	126.6	138.0	128.3
7D	100.4	106.2	102.0
8D	90.6	96.6	93.0
C-1-A6	33.1	36.5	37.4
C-1-A7	38.9	36.5	37.4
C-1-B2	34.9	38.1	39.1
C-1-B3	41.4	38.1	39.1
C-1-B15	46.5	40.7	42.6
2-C-SC	73.7	75.7	72.9
2-C-SD	91.0	90.4	87.3
2-C-SA	68.5	71.8	69.1
2-C-SH	90.8	90.8	86.4

Moreover, the data in Table. 2 can be developed into a new chart and shows trendline from the data plot of each specimen ID.

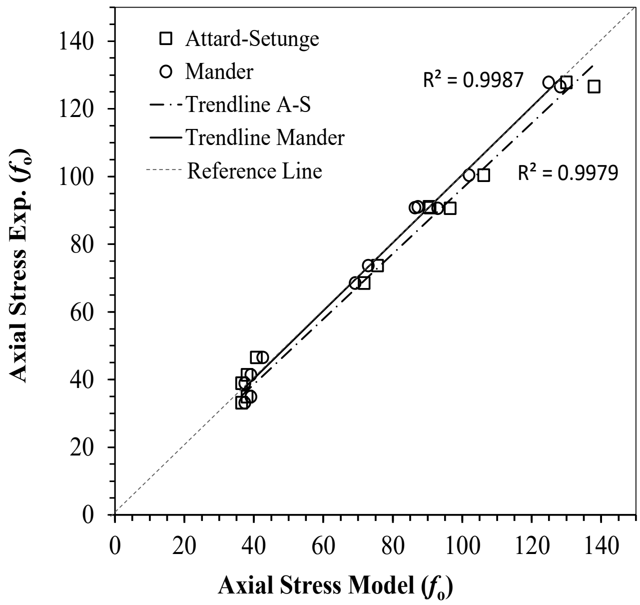


Fig. 9. Comparison between axial peak stress (f_o) from experimental data and both proposed model

Another comparison method by calculating the deviation between each axial stress value (f) at every strain (ε) according to Fig. 2 to Fig. 8. The deviation can be statistically transformed into Root Mean Square Deviation (RMSD) to choose the best model that can be used for another reinforced concrete column analysis.

$$RMSD = \sqrt{\sum_{i=1}^{n} \frac{\left(f_{experimental} - f_{model}\right)^2}{n}} \tag{17}$$

The RMSD value can be obtained by appliying the formulation into every stress-strain relationship in Fig. 2 to Fig. 8. And the conclution of RMSD result can be found in Fig. 10.

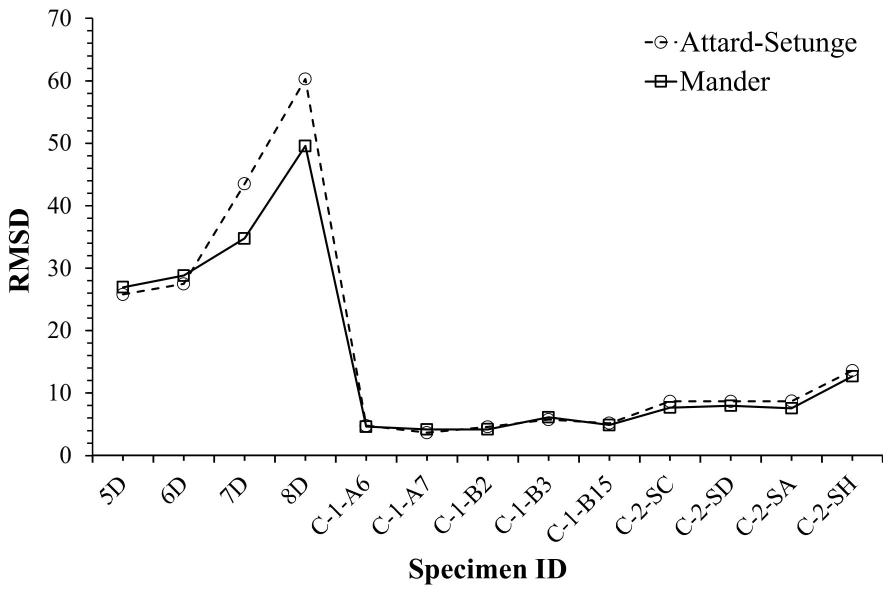


Fig. 10. The value of RMSD from experimental data compared with Attard-Setunge's and Mander's constitutive model.

5. Results and Discussions

According to Fig. 9, there are two types of trendlines that can be compared. The first trendline belongs to the comparison of peak stress (f_o) from Attard-Stunge's [2] model and experimental data. It has an R square value of 0,9979. The second trendline belongs to the comparison of peak stress (f_o) from Mander's et al [1] model and experimental data. It has an R square value of 0,9987. The two R square value are close to 1, so the most suitable method is by comparing both linear regressions with the reference line. According to Fig. 4, the second trendline is the most closely resembles the reference line. It means that the peak stress (f_o) value of the Mander's et al [1] model, from mathematical simulation, has the most similar value with the peak stress (f_o) of the experimental data.

As shown in Fig. 10, there are two RMSD value in every Specimen ID. The dot icon represents the RMSD value obtained from the Attard-Setunge's [2] stress-strain curve compared to the experimental stress-strain curve. And the rectangular icon represents the RMSD value obtained from the Mander's et al [1] stress-strain curve compared to the experimental stress-strain curve. RMSD is commonly used to measure the difference between two sets of data, in this research it is two sets of stress-strain curve. A smaller RMSD value represents a more similar form between two stress-strain curves. Form Fig.10 contain eight specimen ID that have smaller RMSD values obtained from Mander's et al [1] model compared to the experimental data. With a smaller RMSD value, the Mander's et al [1] model represents a more similar form. So that, Mander's et al [1] model is preferable to be used in further research about reinforced concrete rectangular column.

Meanwhile in fig. 10, the specimen ID 5D, 6D, 7D and 8D have a very large RMSD value compared to other specimen ID. This indicates a huge difference forms between the models and the experimental data. This can be caused by the configuration of transverse reinforcement that is too tight as shown on the Table. 1. This affects the behavior of the column to be too ductile and requires a large strain to reach the ultimate axial stress. It can be proven by see Fig.2 and Fig. 3 where the model has an ultimate condition earlier than the experimental results, which have not even reached the ultimate condition.

6. Conclusion

There are many parameters that can be extracted from the results of the fiber-based cross-sectional analysis in MATLAB, such as column interaction diagram; moment-curvature, load-deflection, and stress-strain relationships. However, only the axial stress-strain relationship is obtained from the computational process described in this paper. Because research on the stress-strain relationship constitutive model is the most commonly parameter used by researchers to describe the behavior of concrete. So that in order to analyze the confined concrete behavior, a comparison between the stress-strain model and the existing experimental data is shown. The peak stress (f_o) value from Mander's et al [1] model, from mathematical simulation, has the most similar value with the peak stress (f_o) from experimental data. There are eight specimen ID that have smaller RMSD values obtained from Mander's et al [1] data compared to experimental data. According to the total amount of Specimen ID, it was found that Mander's et al [1] constitutive model has a more similar form compared with Attard-Setunge's [2] constitutive model. Therefore Mander's et al [1] constitutive model is prefered to be used in further analysis problem concerning reinforced concrete column. Furthermore, the use of stress-strain curves as a comparison tool is the begining of another larger research.

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